Semiconductor Devices Summary

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1. Constants & Various

Constants (@300K)

$\varepsilon_0 = 8.854 * 10^{-12}\ F/m \qquad m_0 = 9.11 * 10^{-31}\ kg$

$k = 1.38 * 10^{-23}\ J/K = 8.617 * 10^{-5}\ eV/K$

$\frac{kT}{q} = 0.0259\ V\ , \frac{q}{kT} = 38.61 \frac{1}{V}\ , kT = 25.9\ meV$

$1\ eV = 1.602 * 10^{-19}\ J \qquad q = 1.602 * 10^{-19}\ A\ s$

Silicon (@300K) -> 4 valence electrons

$n_i^2 = 9.3 * 10^{19}/cm^6 \qquad n_i = 9.65 * 10^9/cm^3$

$N_C = 2.86 * 10^{19}/cm^3 \qquad N_V = 2.66 * 10^{19}/cm^3$

$\varepsilon_s = 11.8 * \varepsilon_0 \qquad v_{th} \approx 10^7\ cm/s$

$\varepsilon_{ox} = 3.9\ (SiO_2) \qquad E_G = 1.12\ eV \qquad \chi_S = 4.05\ V$

Quantum physics

$$h = 6.625 * 10^{-34}\ J\ s \quad = \lambda * p$$

$$\omega = \frac{2\pi}{T} = v * k \qquad k = \frac{2\pi}{\lambda} \qquad p = \frac{h}{2\pi} * k = h' * k$$

$$\frac{1}{2}\, m\, v_{th} = \frac{3}{2}\, kT \rightarrow \ v_{th} = \sqrt{3kT/m_0} \approx 10^7\ cm/s$$

Electronics

$$R = \rho * {}^{Length}/_{Area} \quad , \rho\ conductivity\ (\Omega/cm)$$

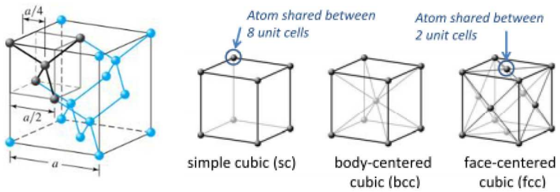
$$E_{kin} + E_{pot} = constant$$

$$V = -\frac{E_{pot}}{q} = -\int E\, dx\ , \qquad E = -\frac{dV}{dx}$$

2. Crystals and Current Carriers

Semi-Conductor: Conductivity controllable over orders of magnitude by means of :
Impurities (doping), light, temperature, EM-fields

Coordination number: number of nearest neighbors



$r = {}^{\sqrt{3}}/_8\ a \qquad r = {}^{1}/_2\ a \qquad r = {}^{\sqrt{3}}/_4\ a \qquad r = {}^{\sqrt{2}}/_4\ a$

Simple Metals: coord. number > # of valence electrons

Transition Metals: bonds covalent-like , harder

Covalent Bonding: hybridization of s- & p-orbitals, stiff

-> *tetrahedral bonding:* coord. number = 4, 8N states

s-orbitals : 2 allowed states ; **p-orbitals** : 6 allowed states

Partially filled/empty bands conduct currents !

Band gap: between valence and conduction band

Intrinsic carriers

No doping, pure semiconductor, created by heat

$$n_0 = p_0 = n_i \quad \sim\ {}^{1}/_{E_G}$$

E_G: *Silicon* $1.12\ eV$, *GaAs* $1.42\ eV$ @ $300\ K$

Extrinsic carriers

Donors (n-type): give electrons (P, As, Sb)
Acceptors (p-type): give holes (B, Al, Ga, In)

Overall, solid is neutral: one fixed charge, one free

$$\boxed{p_0 = {}^{n_i^2}/_{N_D} \quad , \quad n_0 = {}^{n_i^2}/_{N_A}}$$

Fermi Dirac Statistics

F(E): probability of finding an electron with energy E

$$F(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \cong e^{-\frac{(E-E_F)}{kT}} \qquad E \gg E_F$$

Fermi level E_F : energy where $F(E = E_F) = {}^{1}/_{2}$

Probability of finding a hole: $H(E) = 1 - F(E)$

$$n_0 = \int_{E_C}^{\infty} f(E)\ x\ D(E_{kin})\, dE_{kin}\ , p_0 = \int_{-\infty}^{E_V} (1 - f(E))\ x\ D(E_{kin})\, dE_{kin}$$

$$Density\ of\ State: \quad D(E_{kin}) = \frac{8\pi\sqrt{2}}{h^3} (m^*)^{3/2}\ (E_{kin})^{1/2}$$

$$Kinetic\ energy: \quad E_{kin} = \frac{|\vec{p}|}{2m^*} = \frac{(p_x)^2 + (p_y)^2 + (p_z)^2}{2m^*}$$

i) Fermi levels in all regions will lign up

ii) Far away from transition, Fermi level is like without junction (material doesn't "know")

iii) At Equilibrium/Steady-State, E_F must be flat (constant) so that no current will be flowing

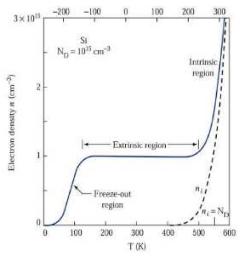
Carrier concentration

$$N_C = \frac{4\sqrt{2}(\pi m^* kT)^{3/2}}{h^3}, \qquad N_V = \frac{4\sqrt{2}(\pi m^* kT)^{3/2}}{h^3}$$

$$\boxed{\begin{aligned} n_0 &= N_C * e^{-\frac{E_C - E_F}{kT}} = n_i * e^{\frac{E_F - E_i}{kT}} \\ p_0 &= N_V * e^{-\frac{E_F - E_V}{kT}} = n_i * e^{\frac{E_i - E_F}{kT}} \end{aligned}}$$

$$n_i^2 = N_V * N_C * e^{-{}^{E_G}/_{kT}}\ , \qquad E_G = E_C - E_V$$

Constant product: $n_0 * p_0 = n_i^2$



Carrier "Freeze-Out": T << 0°C

"Extrinsic Region": donors ionized

"Intrinsic Region": doping irrelevant

3. Carrier transport

Diffusion current: concentration gradients

from *high to low* concentration

$$J_n = q\, D_n \frac{dn}{dx} \quad , \qquad J_p = -q\, D_p \frac{dp}{dx}$$

Drift current: electric field

holes with field, electrons against it

$$J_n = n\, q\, \mu\, \vec{E} \quad , \qquad J_p = p\, q\, \mu\, \vec{E}$$

Total current:

$$J_n = nq\mu\vec{E} + qD_n \frac{dn}{dx} \, , \qquad J_p = pq\mu\vec{E} - qD_p \frac{dp}{dx}$$

Conductivity

$$J_{drift,tot} = \sigma\, E \qquad \rightarrow \qquad \sigma = nq\mu_n + pq\mu_p$$

Einstein relation

$$D_n = \frac{kT}{q}\, \mu_n, \qquad D_p = \frac{kT}{q}\, \mu_p$$

In PN Junction: only diffusion currents (flat bands)

$$\frac{dn}{dx} = \frac{n_{po}(e^{qV_F/kT} - 1)}{L_n} \, , \frac{dp}{dx} = \frac{p_{no}(e^{qV_F/kT} - 1)}{L_p}$$

$$J_t = q\, D_n * \frac{dn}{dx} - q\, D_p \frac{dp}{dx} = J_S * (\, e^{qV_F/kT} - 1)$$

$$\boxed{J_t = \left[\frac{q\, D_n\, n_{po}}{L_n} + \frac{q\, D_p\, p_{n0}}{L_p}\right] * (e^{qV_F/kT} - 1)}$$

Reverse breakdown

i) *Band-to-Band Tunneling (Zener)*

applies when both sides are heavily doped

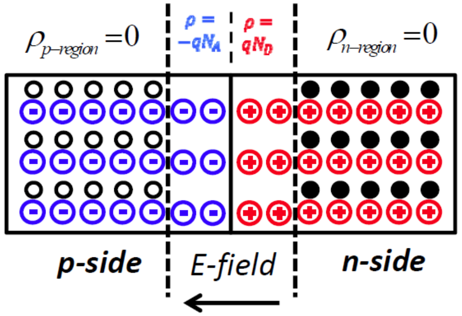
ii) *Avalanche Multiplication*

strong electric field creates large kinetic energy to the carriers, so that they ionize others via collision

4. PN Junction

Built-in voltage

Band-bending that balances drift & diffusion currents



$$\boxed{V_{bi} = \frac{kT}{q} * \ln\left(\frac{N_A * N_D}{(n_i)^2}\right) = \frac{1}{2}\, E_{max} * W}$$

Forward Bias: reduce band bending, less difference

more minority carriers -> minority carrier injection

Reverse Bias: increase band bending, less minority

Band-bending = presence of an electric field

Conduction Band Edge: E_{pot} of electrons

Valence Band Edge: E_{pot} of holes

Diode currents: minority carriers

$$n_{p0} = \frac{(n_i)^2}{N_A} = N_D * e^{-\frac{qV_{bi}}{kT}}$$

$$p_{no} = \frac{(n_i)^2}{N_D} = N_A * e^{-\frac{qV_{bi}}{kT}}$$

Under Forward-Bias: **Shockley Boundary Conditions**

$$\boxed{\begin{aligned} n_p &= N_D * e^{-\frac{q(V_{bi}-V_F)}{kT}} = n_{po} * e^{+\frac{qV_F}{kT}} \\ p_n &= N_A * e^{-\frac{q(V_{bi}-V_F)}{kT}} = p_{no} * e^{+\frac{qV_F}{kT}} \end{aligned}}$$

Poisson Equation

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_r * \varepsilon_0} = \frac{\rho}{\varepsilon_S} \quad \rightarrow \quad V_{bi} = -\int_{-x_p}^{x_n} E(x)\; dx$$

Depletion approximation

$$|E_{max}| = |E(x=0)| = \frac{qN_A x_p}{\varepsilon_S} = \frac{qN_D x_n}{\varepsilon_S}$$

$$W = x_p + x_n = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(\,V_{bi} - V_{apply})}$$

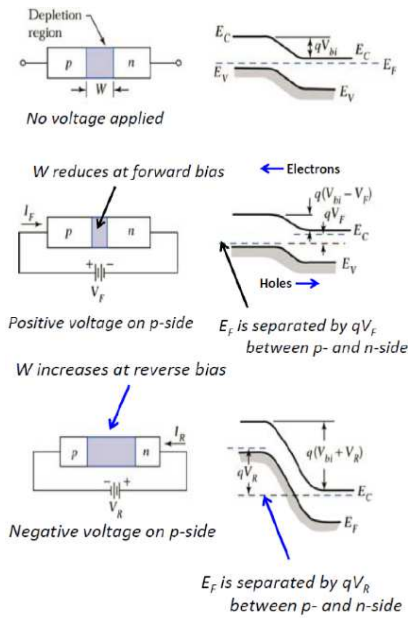
Neutrality: $N_A\, x_p = N_D x_n$ (same areas)

One-Sided junction: only depletion on lightly-doped side

$$W \approx \sqrt{\frac{2\,\varepsilon_S}{q}\frac{1}{N_D} V_{bi}} \quad , \qquad N_D \ll N_A$$

Depletion capacitance

$$C_j = \frac{dQ}{dV} = \frac{\varepsilon_S}{W(\,V_{bi}, V_{apply})} = \sqrt{\frac{q\, \varepsilon_0 \varepsilon_r\, N_A\, N_D}{2\, V_{bi}\, (N_A + N_D)}}$$



4. Generation and Recombination

Recombination brings the system back to equilibrium

Non-equilibrium concentration:

$$n = n_0 + \Delta n\, ,p = p_0 + \Delta p\, , \quad \Delta n = \Delta p$$

Recombination rate (even at Non-equilibrium):

$$R = \beta * (n * p)$$

Thermal generation rate

$$G_{th} = R_{th} = \beta * (\, n_{n0} * p_{n0})$$

External excitation (e.g. Light) gives additional term:

$$G = G_L + G_{th} \;\rightarrow\; \frac{dp_n}{dt} = G_L + G_{th} - R$$

Direct recombination

Direct recombination across the bandgap results in the emission of a photon with energy $E_G = h * f$

Net generation rate U

$$U = \beta * (\, n * p - n_i^2\,) = G_L = R - G_{th}$$

Under low-level injection: $p_{n0} \ll n_{n0}\, ,\Delta p \ll n_{n0}$

$$U = \frac{\Delta p}{\tau_p} \quad , \qquad \tau_p = \frac{1}{\beta\, n_{n0}}$$

τ : Minority carrier lifetime (how fast decay)

Example: Lesson 5, p.7

- Light ON

$$G_L = U = \frac{p_n - p_{n0}}{\tau_p} \;\rightarrow\; p_n = p_{no} + \tau_p G_L$$

- Light OFF:

$$G_L = 0 \rightarrow \frac{dp_n}{dt} = G_{th} - R = -\frac{p_n - p_{n0}}{\tau_p}$$

$$\rightarrow\; p_n(t) = p_{n0} + \tau_p\, G_L\, e^{-t/\tau_p}$$

Indirect recombination (Neamen: p.223)

G-R Centers in the Gap (defect states near midgap)
These “traps” facilitate the return of an electron
G/R centers: most effective if E_t near intrinsic E_i

$$U \approx v_{th}\sigma_0 N_t * \frac{\Delta p}{1 + \left(\frac{2n_i}{n_{n0}}\right)\cosh\left(\frac{E_t - E_i}{kT}\right)} = \frac{\Delta p}{\tau_p}$$

N_t : Density of Recombination Centers
σ : Recombination Center cross section

$e_n = v_{th}\sigma_n n_i e^{(E_t - E_i)/kT}$ Electron emission prob.
$e_p = v_{th}\sigma_p n_i e^{(E_i - E_t)/kT}$ Hole emission probability

$R_a = nN_t(1 - f) * v_{th}\sigma_n$ Electron capture rate
$R_b = e_n N_t f$ Electron emission rate
$R_c = pN_t(f) * v_{th}\sigma_p$ Hole capture rate
$R_d = e_p N_t\,(1 - f)$ Hole emission rate

Surface recombination: “dangling bonds” at surface

Continuity equation

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx} + (G_n - R_n)\, , \frac{dp}{dt} = -\frac{1}{q}\frac{dJ_p}{dx} + (G_p - R_p)$$

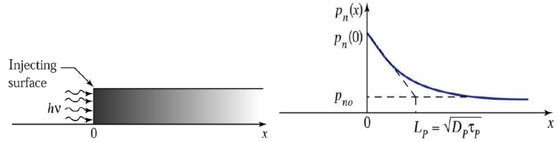
$$\boxed{\begin{aligned} \frac{dn_p}{dt} &= n_p\mu_n\frac{d\vec{E}}{dx} + \mu_n\vec{E}\frac{dn_p}{dx} + D_n\frac{d^2 n_p}{dx^2} + G_n - \frac{n_p - n_{p0}}{\tau_n} \\ \frac{dp_n}{dt} &= -p_n\mu_p\frac{d\vec{E}}{dx} - \mu_p\vec{E}\frac{dp_n}{dx} + D_p\frac{d^2 p_n}{dx^2} + G_p - \frac{p_n - p_{n0}}{\tau_p} \end{aligned}}$$

Steady State: Quantities are Time Independent
Zero Field: fields in neutral regions are approx. zero

Generation:	deficiency of minority carriers
Recombination:	excess of minority carriers

Exp: Steady State surface Generation

Long diode: **semi-infinite, exponential decay** $\boldsymbol{L \ll W}$



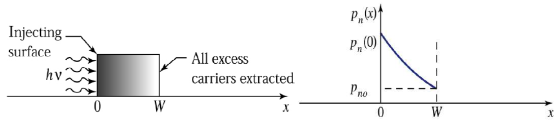
$$p_n(0) = const, \qquad p_n(x \rightarrow \infty) = p_{n0}$$

$$p_n(x) = p_{n0} + [\, p_n(0) - p_{n0}] * e^{-x/L_p}$$

$$J_p(x_n) = -qD_p\left[\frac{dp_n}{dx}\right]_{x_n} = \frac{qD_p p_{n0}}{L_p}\,(e^{\frac{qV_F}{kT}} - 1)$$

$$J_n(-x_p) = qD_n\left[\frac{dn_p}{dx}\right]_{-x_p} = \frac{qD_n n_{p0}}{L_n}\,(e^{\frac{qV_F}{kT}} - 1)$$

Short diode: **finite, linear decay** $\boldsymbol{L \gg W}$



$$p_n(0) = const, \qquad p_n(x = W) = p_{n0}$$

$$p_n(x) = p_{n0} + [p_n(0) - p_{n0}]\left[\frac{\sinh\left(\frac{W - x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right]$$

Minority Carrier Diffusion Length:

$$L_p = \sqrt{D_p\,\tau_p}$$

Quasi-Fermi Levels

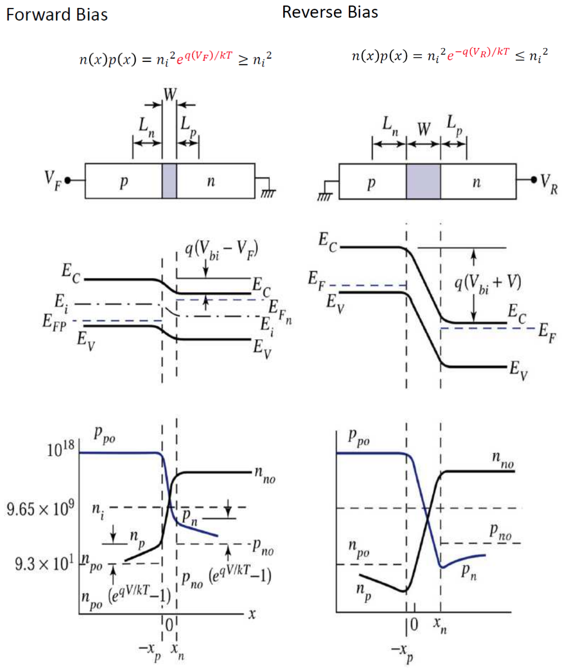
Under bias, the equilibrium Fermi level splits into 2 distinct quasi-Fermi levels that describe carrier statistics in each diode region

$$n(x) = N_C\, e^{-(\,E_C(x) - E_{Fn})/kT}\, , \quad p(x) = N_V e^{-\left(E_{Fp} - E_V(x)\right)/kT}$$

$$n(x)p(x) = N_C N_V\, e^{-\frac{E_G}{kT}}\, e^{(E_{Fn} - E_{Fp})/kT}$$

$$E_{Fn} - E_{Fp} = q\, V_F$$

Carrier Profile through Depletion Region



Capacitance in depletion region

Depletion capacity per unit square [$F\ /\ cm^2$]

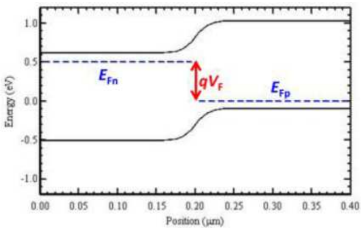
$$C_A = \frac{C}{A} = \frac{\varepsilon_0 \varepsilon_r}{W}\ , \qquad W: depletion\ width$$

Non idealities

$$n(x) = N_C\ e^{-\frac{E_C(x) - E_{Fn}}{kT}}$$

$$p(x) = N_V\ e^{-\frac{E_{Fp} - E_V(x)}{kT}}$$

$$n(x)\ p(x) = N_C\ N_V\ e^{-\frac{E_G}{kT}} * e^{\frac{E_{Fn} - E_{Fp}}{kT}} = n_i^2\ e^{\frac{qV_F}{kT}}$$



Generation currents

Reverse bias

Carrier Deficit -> Generation current

$$J_{gen} = \int_0^W qG\ dx = \frac{q\ n_i}{\tau_g}\ W \quad , \qquad G = \frac{n_i}{\tau_g}$$

$$J_{RT} = J_S + J_{gen} = \left[\frac{qD_n}{N_A L_n} + \frac{qD_p}{N_D L_p}\right] n_i^2 + \frac{q\ W\ n_i}{\tau_g}$$

Forward bias

Carrier Excess -> Recombination current

$$U_{max} = \sigma_0 N_t \frac{n_i^2 (e^{\frac{qV_F}{kT}} - 1)}{p_n + n_n + 2n_i} \approx \frac{1}{2} v_{th} \sigma_0 N_t n_i e^{qV_F/kT}$$

$$J_{rec} = \int_0^W q\ U\ dx = \frac{q\ W\ n_i}{2\ \tau_r}\ e^{\frac{qV_F}{2kT}}$$

$$J_{FT} = \left[\frac{qD_N}{N_A L_n} + \frac{qD_p}{N_D L_p}\right] n_i^2\ e^{qwV_F/kT} + \frac{q\ W\ n_i}{2\ \tau_r}\ e^{qV_F/2kT}$$

Ideality Factor η: characterizes Diode Forward Current Ideality

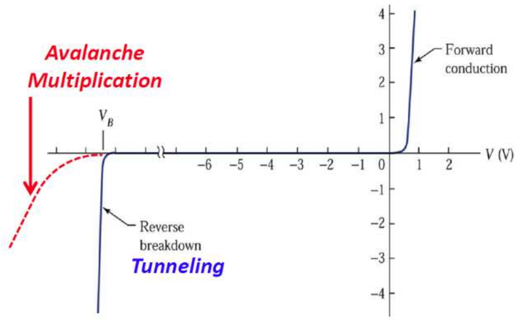
Materials with longer recombination lifetime have better ideality

$$J_{FT} = J_S\left(e^{\frac{qV_F}{kT}} - 1\right) + J_{rec} \sim \exp\left[\frac{q\ V_F}{\eta\ kT}\right]$$

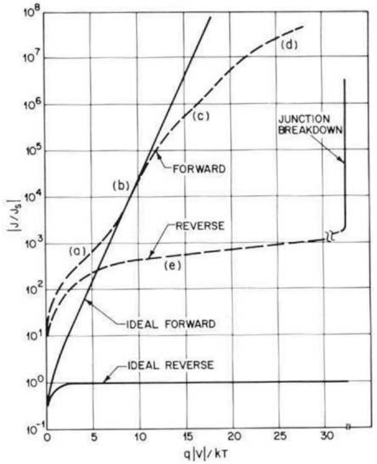
Reverse Breakdown of Diode:

i) *Band-to-Band Tunneling (Zener)*
applies when both sides are heavily doped

ii) *Avalanche Multiplication*
strong electric field creates large kinetic energy to the carriers, so that they ionize others via collision



Real PN Junction Diode



a) Recombination in Deplention Region

b) Ideal Injection ($\eta = 1\ ;\ {}^{60mV}/_{dec}$)

c) High-Level Injection ($\eta \rightarrow 2$)

d) Series resistance effects (ohmic loss)

e) Generation in Deplention region

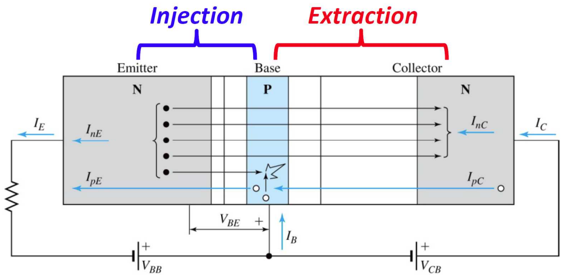
Ohmic losses

Ohmic losses reduce the internal voltage that actually appears across the depletion; at low current levels negligible

$$I \approx I_S \frac{e^{qV_A/kT}}{e^{qIR/kT}}$$

5. Bipolar Junction Transistor (BJT)

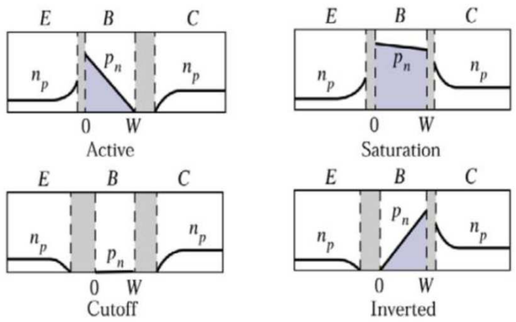
BJT is a *Minority Carrier Device* and acts as an *ideal current source* ($I_{Collector}$ does not vary with V_{CB})

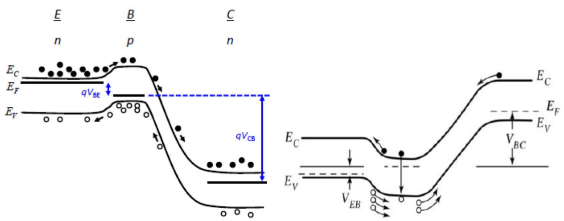


Emitter/Base Junction (in active mode)
Forward-Biased: **Minority Carrier Injection**

Base/Collector Junction (in active mode)
Reversed-Biased: **Minority Carrier Extraction**

Modes	B-E	B-C
Active	Forward	Reverse
Saturation	Forward	Forward
Cutoff	Reverse	Reverse
inverted	Reverse	forward

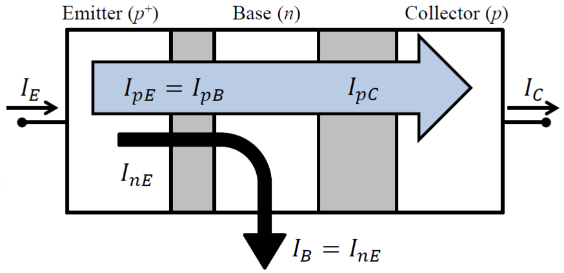


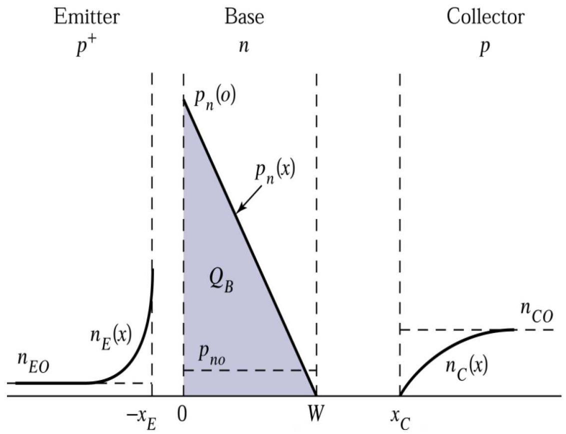


NPN **PNP**

Ideal currents

- Injection from Emitter into Base
- No Generation/Recombination in Base Layer
- neglect Diode Leakage Current





Constant carrier densities in the depleted regions
Assumed no recollection or generation

$$n_E = n_{E0}, x \to \infty\,;\; n_E(-x_E) = n_{E0} * e^{\frac{q\,V_{BE}}{kT}}$$

$$n_E(x) = n_{E0} + (\,n_E(-x_E) - n_{E0}) * e^{\frac{x + x_E}{L_E}}$$

$$\to I_{nE} = I_B = q\,D_{nE}\frac{dn_E}{dx} = \frac{qD_{nE}}{L_{nE}}\,n_{E0}\left(e^{\frac{qV_{EB}}{kT}} - 1\right)$$

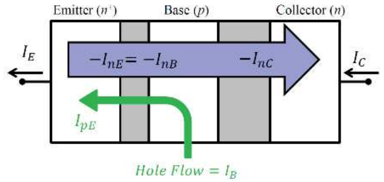
$$p_B(0) = p_{B0} * e^{\frac{qV_{BE}}{kT}}\,, \qquad p_B(W) = p_{B0} * e^{\frac{qV_{BC}}{kT}}$$

$$p_B(x) = p_B(W) + (\,p_B(0) - p_B(W)) * \left(1 - \frac{x}{W}\right)$$

$$\to I_{pB} = -q\,D_{pB}\frac{dp_{nB}}{dx} = \frac{qD_{pB}}{W_B}\,p_{B0}\left(e^{\frac{qV_{EB}}{kT}} - e^{\frac{qV_{BC}}{kT}}\right)$$

For NPN:

$$\beta = \frac{D_{nB}}{D_{pE}}\frac{L_{pE}}{W_B}\frac{N_{DE}}{N_{AB}}$$



$$I_B = I_{pB} = I_{pE} = -\frac{q\,D_P}{L_{pE}}\,p_{E0}\left(e^{\frac{q\,V_{BE}}{kT}} - 1\right)$$

$$I_C = I_{nB} = -\frac{q\,D_{NB}}{W_B}\,n_{B0}\left(e^{\frac{q\,V_{BE}}{kT}} - e^{\frac{q\,V_{BC}}{kT}}\right)$$

Common Emitter current gain

$$\beta = \frac{I_C}{I_B} = \frac{I_{pC}}{I_{nE}} = \frac{I_{pE}}{I_{nE}} = \frac{D_{pB}}{D_{nE}}\frac{L_{nE}}{W}\frac{N_{AE}}{N_{DB}}, \qquad V_{CB} = 0$$

$$\boxed{I_C = \alpha\,I_E = \beta\,I_B\,, \qquad \beta = {}^{\alpha}/_{1-\alpha}}$$

Emitter doping must be higher than base doping:

$$I_{pC} \gg I_{nE} \quad \Leftrightarrow \quad N_{AE} \gg N_{DB}$$

Doping Ration most powerful factor to reach gain

Gummel-Characteristics: ${}^{60mV}/_{dec}$ $gain\ in\ I/V$

Transconductance

Collector current: $I_C = I_S * \left(e^{\frac{qV_{EB}}{kT}} - 1\right)$

Transconductance: $g_m \approx \frac{I_C}{kT\,/\,q}$

Non-ideal currents

NPN	PNP
$J_{nE} = -qD_{nB}\frac{\partial n_{pB}}{\partial x}\Big\vert_{x=0}$	$J_{nE} = -qD_{nE}\frac{\partial n_{pE}}{\partial x}\Big\vert_{x=-x_E}$
$J_{pE} = -qD_{pE}\frac{\partial p_{nE}}{\partial x}\Big\vert_{x=-x_E}$	$J_{pE} = -qD_{pB}\frac{\partial p_{nB}}{\partial x}\Big\vert_{x=0}$
$J_{nC} = qD_{nB}\frac{\partial n_{pB}}{\partial x}\Big\vert_{x=W}$	$J_{nC} = qD_{nC}\frac{\partial n_{pC}}{\partial x}\Big\vert_{x=x_C}$
$J_{pC} = -qD_{pC}\frac{\partial p_{nC}}{\partial x}\Big\vert_{x=x_C}$	$J_{pC} = -qD_{pB}\frac{\partial p_{nB}}{\partial x}\Big\vert_{x=W}$
$J_{BB} = J_{nE} - J_{nC}$	$J_{BB} = J_{pE} - J_{pC}$

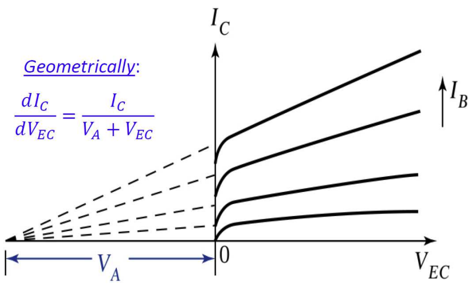
No base recombination, if:

$W_B \ll L_{nB} \Rightarrow J_{BB} \approx 0$	$W_B \ll L_{pB} \Rightarrow J_{BB} \approx 0$
$I_E = I_{pE} + I_{nE}$	$I_E = I_{nE} + I_{pE}$
$I_C = I_{pC} + I_{nC}$	$I_C = I_{nC} + I_{pC}$
$I_B = I_{pE} + (I_{nE} - I_{nC}) - I_{pC}$	$I_B = I_{nE} + (I_{pE} - I_{pC}) - I_{nC}$

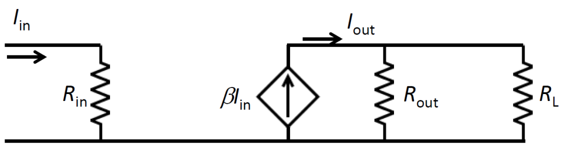
"Early" – effect:

In practice, the I_C depends on V_{BC}. The depletion region becomes wider with increasing BC reverse bias, decreasing the undepleted base width, which increases I_C. To avoid this, the base doping must be higher than collector:

$$N_{DB} \gg N_{AC}$$



Power Gain from Amplifier



$$G = \frac{I_{out}^2}{I_{in}^2}\frac{R_{load}}{R_{in}} = \beta^2 \frac{R_L}{R_{in}} = \beta \frac{R_L I_C}{kT\,/\,q}$$

For power gain: $R_{out} \to \infty \quad \Leftrightarrow \quad V_A \to \infty$

$$G_A = \beta^2 \frac{R_L}{R_{in}} \left(\frac{[V_A + V_{CE}]/I_C R_L}{[V_A + V_{CE}]/I_C R_L \ + 1} \right)$$

Used power: $P = V_{CE} * I_C$

Cost of power gain: $P_D = V_{CE} * I_C - P_{out}$

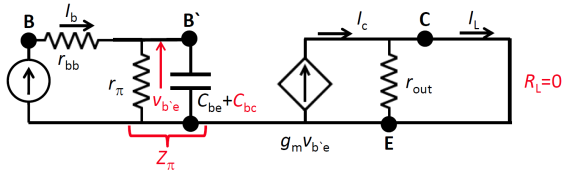
Maximum Power gain (with impedance matching)

$$G_p = \frac{1}{f^2}\frac{f_T}{8\pi R_B C_{BC}}, \qquad f_{MAX} = \sqrt{\frac{f_T}{8\pi R_B C_{BC}}}$$

Cutoff frequency

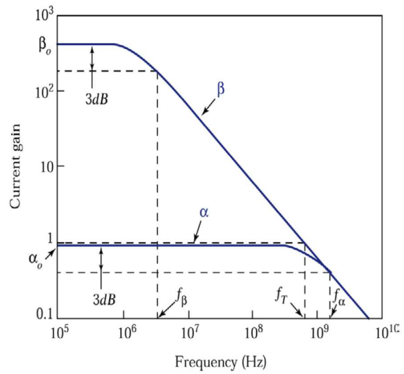
Unity Current Gain Cutoff frequency: $\beta(\,f_T\,) = 1$

Measured with Short-Circuit load ($R_L = 0$)



$$h_{fe}(\omega) = \frac{I_C}{I_B} = \frac{g_m r_\pi}{1 + j\omega\, r_\pi\, C_\pi} = \beta(\omega)$$

$$h_{fe}(\omega = 0) = g_m r_\pi$$



Transistor behaves as a Low-Pass

$$\beta(\omega) = h_{fe}(\omega) = \frac{\beta_0}{1 + j\,(\,f\,/\,f_\beta\,)}$$

$$f_\beta = \frac{1}{2\pi\, C_\pi\, r_\pi}, \qquad \beta_0 = g_m\, r_\pi$$

Cutoff Frequency: $f_{To} = \dfrac{g_m}{2\pi\, C_\pi}$

Common-Base (BC) Current Gain

$$\alpha(\omega) = \frac{\beta(\omega)}{\beta(\omega) + 1} = \frac{I_C}{I_E} = \frac{\alpha_0}{1 + j\,(\,f\,/\,f_\alpha)}$$

$$f_\alpha = (\,\beta_0 + 1)\, f_\beta, \qquad \alpha_0 = \beta_0 / \beta_0 + 1$$

Additional Delay terms

τ_B : **Base Transit Time** (diffusion across the base)

τ_C : **Collector Signal Delay Time** (through depletion)

$$\tau_B = \frac{Q_B}{J_C} = \frac{W_B^2}{2D_n}, \qquad \tau_C = \frac{W_C}{2 * v_{sat}}$$

$$\frac{1}{f_{\alpha\tau}} = \frac{1}{f_\alpha} + \frac{1}{f_\tau}, \qquad f_\tau = \frac{1}{2\pi\tau}$$

Cutoff frequency including delay terms

$$f_T = \sqrt{\beta_0^2 - 1}\, f_{\beta\tau} \approx \alpha_0\, f_{\alpha\tau} = \frac{1}{2\pi\, \tau_T}$$

Where τ_T **is the transit time** / sum of all delays

$$\tau_T \approx \frac{C_\pi}{g_m} + \tau = \frac{C_\pi}{g_m} + \tau_1 + \tau_2 + \ \ldots$$

Common-emitter delay term: C_π / g_m

Kirk-Effect ("Base spreading")

At high currents, the electron density n_C in the collector becomes comparable to the donor density (npn BJT). Therefore, it cannot be neglected for the calculation of the E-Field in the collector:

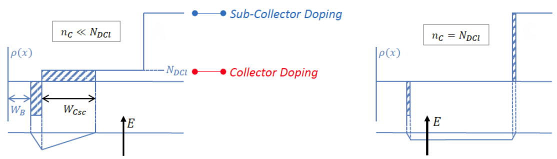
$$E(x) = \frac{q}{\varepsilon_S}\left[\,(N_{DC} - n_C)x + E_{depletion}\,\right]$$

Base spread (increases τ_B , reduces β)

$$W_k = W_C * \left[1 - \sqrt{\frac{V_{CB}/V_{Cd}}{(n_C/N_{DCl}) - 1}}\right]$$

Kirk effect threshold current

$$J_K = q * v_{sat} * N_{DCl}\left(1 + \frac{2\,\varepsilon_S\, V_{CB}}{q\, N_{DCl}\, W_C^2}\right)$$



Base drift field

Carrier transport across the base can be aided by introducing an electric field, such as by **non-flat base doping profiles / grading the doping**.

P-type base with width W_B with an electric field:

$$n_B(x) = -\frac{J_n\, W_B}{q\, D_n} \frac{1 - e^{-\eta\left(1-\frac{x}{W_B}\right)}}{\eta}, \qquad \eta = \frac{W_B}{x_0}$$

η : accelerating field factor / grading

$$\tau_B = \frac{W_B^2}{D_n}\left(\frac{\eta - 1 + e^{-\eta}}{\eta^2}\right), \qquad \tau_B(\eta = 0) = \frac{W_B^2}{2D_{nB}}$$

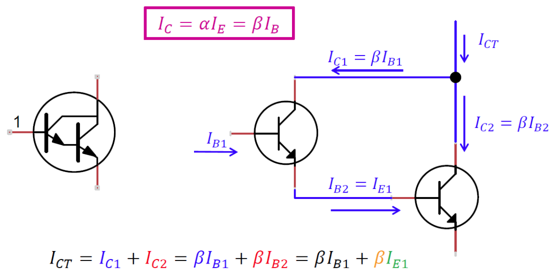
NPN base charge: $Q_B = \int_0^{W_B} -q\, n_B\, dx \quad [\,C/cm^2]$

Heterojunction Bipolar Transistor (HBT)

Different materials and bandgaps for emitter & base

$$\beta = \beta_{BJT} * \frac{n_{iB}^2}{n_{iE}^2} = \beta_{BJT}\, e^{\frac{\Delta E_G}{kT}}$$

Darlington Pair



$$I_{CT} = I_{C1} + I_{C2} = \beta I_{B1} + \beta I_{B2} = \beta I_{B1} + \beta I_{E1}$$

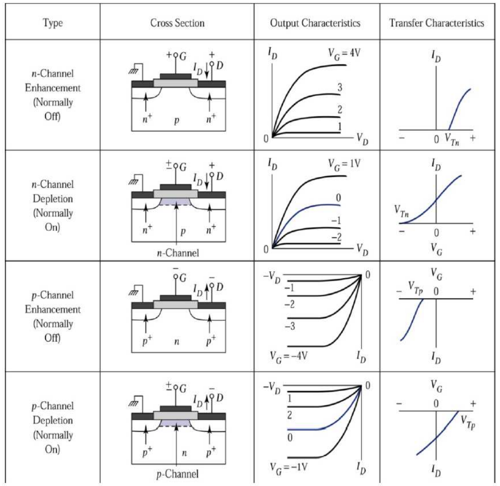
$$I_{CT} = \beta I_{B1} + \beta\, \beta I_{B1}/\alpha = \beta I_{B1} + \beta(\beta+1) I_{B1} = \beta(\beta+2) I_{B1}$$

$$\boxed{\beta_{Darlington} \approx \beta^2}$$

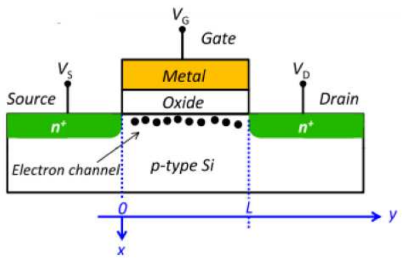
6. MOSFET

In contrast to BJTs majority devices (majority carrier)
N-Channel: electrons, P-Channel: holes

Depletion Mode: channel present at equilibrium
Enhancement Mode: no channel at equilibrium



Structure



Drain-Source voltage V_{DS} : low for uniform channel
Gate-Source voltage V_{GS} : large enough for channel

Channel is built by minority carriers between S & D

Sheet resistance

$$R_S = \rho * \frac{Length}{Area} = \frac{\rho}{Thickness} \quad [\,\Omega/m^2\,]$$

$$R_S(x) = \frac{1}{\mu_n\, C_{OX}\,(V_{GS} - V_T - V(x))}$$

V(x): channel voltage; $V(0) = V_S = 0,\ V(L) = V_{DS}$
V_T : threshold voltage for strong inversion

Basic characteristics

Inversion layer has thickness X, charge density Q_n

$$Q_n = -q\, n\, X = -C_{OX}\,(V_{GS} - V_T - V(x))$$

$$X = \frac{C_{OX}\,(V_{GS} - V_T - V(x))}{q\, n}, \qquad Z: width$$

$$I_{CH} = I_D = \frac{\mu_n\, C_{OX}}{2} \frac{Z}{L} [\,2(V_{GS} - V_T)\, V_{DS} - V_{DS}^2\,]$$

Pinch-Off

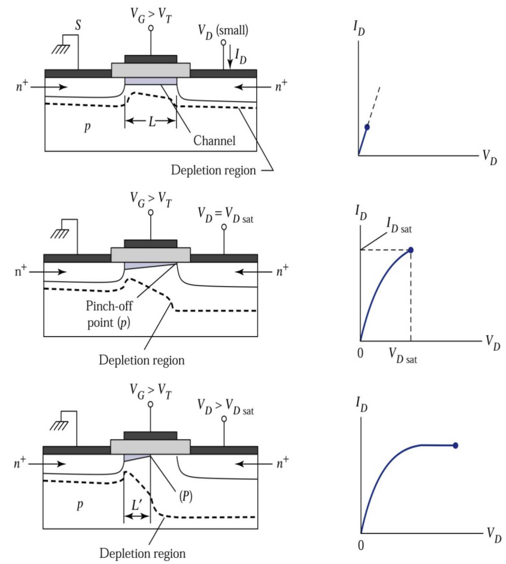
Pinch-off when $V_{DS} \geq V_{GS} - V_T$ at drain side

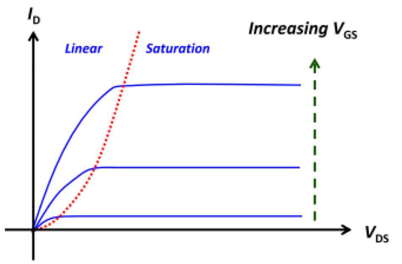
Linear region: $V_{DS} < V_{GS} - V_T$

$$I_{CH} = I_D = \frac{\mu_n\, C_{OX}}{2} \frac{Z}{L} [\,2(V_{GS} - V_T)\, V_{DS} - V_{DS}^2\,]$$

Saturation region: $V_{DS} \geq V_{GS} - V_T$

$$I_{D_{Sat}} = \frac{\mu_n\, C_{OX}}{2} \frac{Z}{L} (\,V_{GS} - V_T\,)^2$$

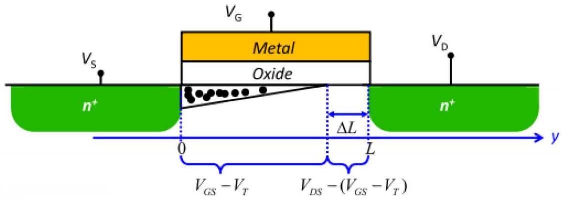




Transconductance in Saturation region

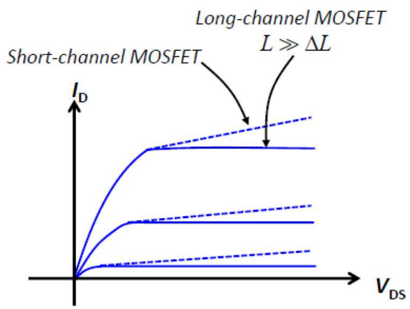
$$g_m = \frac{dI_{D_{Sat}}}{dV_{GS}} = \frac{\mu_n C_{OX}\, Z}{L}\,(V_{GS} - V_T)$$

Channel length modulation (L12P2)



Assume $\Delta L \ll L$: channel length independent of V_{DS}

When we cannot assume that $\Delta L \ll L$, we have a **short-channel MOSFET** whose drain-current increases with increasing V_{DS} ! Like Early for BJTs

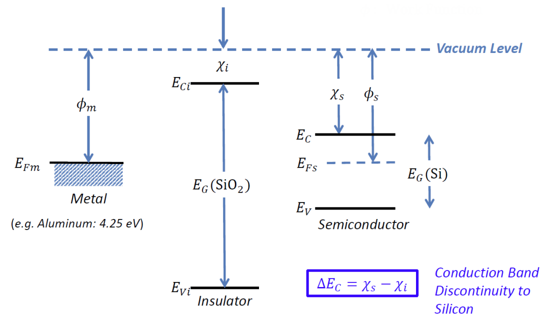


Reducing the channel length increases:

- *transconductance* g_m
- operation speed
- device density

But V_T decreases (threshold voltage shift)

Band diagramm

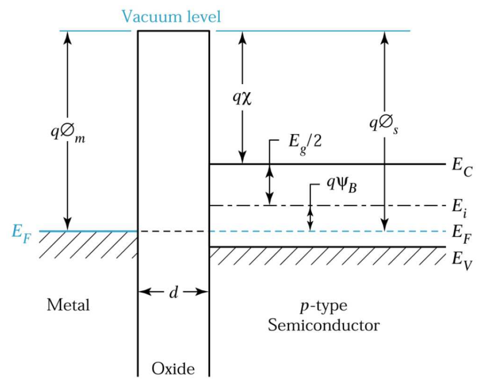


$$\textit{Electron affinity}: \quad \chi = E_0 - E_C \quad [eV]$$
$$\textit{Work function} \quad : \quad \Phi = E_0 - E_F \quad [eV]$$
$$\textit{Vacuum level} \quad : \quad E_0$$

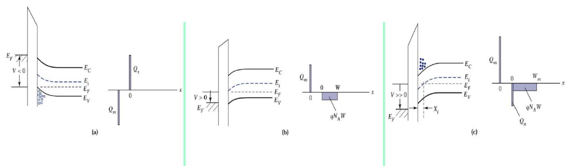
At Equilibrium, the Fermi Level must be constant !

As the metal workfunction differs from the semi-conductor workfunction, there will be bandbending

Flatband voltage: Gate voltage that makes them flat



Channel Modulation



Accumulation (p-Substrate)

Depletion (Negative Space Charge)

Inversion (Electron Channel in p-Substrate)

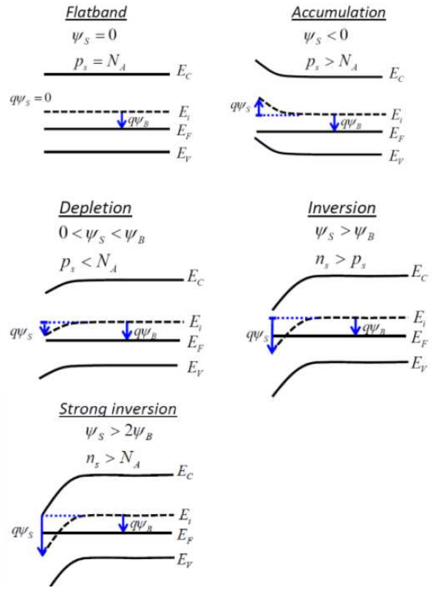
Band bending

General potential: $q\,\Psi(x) = E_i - E_i(x)$

Bulk potential : $q\,\Psi_B(x) = E_i - E_F$

$$\Psi_B = \frac{kT}{q}\ln\left({}^{N_A}/_{n_i}\right)$$

$$p_p = n_i\, e^{\frac{E_i - E_F}{kT}} = n_i\, e^{\frac{q(\Psi_B - \Psi)}{kT}}$$
$$n_p = n_i\, e^{\frac{E_F - E_i}{kT}} = n_i\, e^{\frac{q(\Psi - \Psi_B)}{kT}}$$



Midgap: $\Psi_S = \Psi_B$, $\quad n_p = p_p = n_i$

Depletion region width

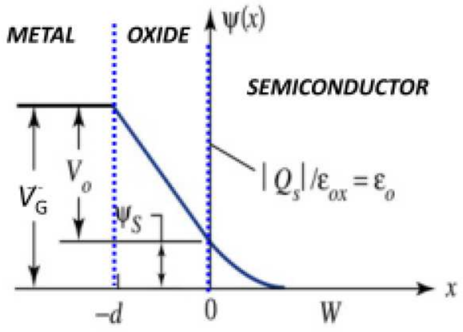
$$\Psi(x) = \Psi_S\left(1 - \frac{x}{W}\right)^2, \qquad \Psi_S = \frac{qN_A W^2}{2\varepsilon_s}$$

Depletion width: $W = \sqrt{\frac{2\,\varepsilon_S\,\Psi_S}{q\,N_A}}$

Max. at *inversion*: $W_{max} = \sqrt{\frac{2\,\varepsilon_S\,2\,\Psi_B}{q\,N_A}}$, $\Psi_S = 2\Psi_B$

Electric field: $E_s(x) = \frac{q\,N_A}{\varepsilon}\,(W - x)$

Ideal gate voltage relationship



$$V_G = V_{ox} + \Psi_S = d * E_{ox} + \Psi_S$$

V_G : Potential drop across oxide & semiconductor

$$V_{ox} = \frac{\sqrt{2\, q\, \varepsilon_S\, N_A\, \Psi_S}}{C_{OX}}, \qquad C_{OX} = \frac{\varepsilon_{ox}}{d}$$

$$V_G = \frac{\sqrt{2\, q\, \varepsilon_S\, N_A\, \Psi_S}}{C_{OX}} + \Psi_S$$

Threshold voltage ($\Psi_S = 2\,\Psi_B$)

$$V_T = \frac{\sqrt{2\, q\, \varepsilon_S\, N_A\, 2\,\Psi_B}}{C_{OX}} + 2 * \Psi_B$$

After that, W is maximal and stays more or less

Non-ideal gate voltage relationship (*voltage shift*)

Bands are not flat due to

1. Workfunction difference $\Psi_{ms} = (\Phi_m - \Phi_S)\,/\,q$
2. Fixed charges inside the oxide

$$\rightarrow \quad V_G = V_G' + V_{FB}\,, \qquad V_G' : ideal\ gate\ voltage$$

$$V_{FB} = \Psi_{ms} - \frac{1}{\varepsilon_S}\int_{oxide} x\ \rho_{ox}(x)\, dx\ , \qquad \varepsilon_S = C_o\, d$$

If the charge in the oxide is fixed:

$$V_{FB} = \Psi_{ms} - \frac{Q_O}{C_{ox}}\,, \qquad Q_O\ \left[\frac{C}{cm^2}\right]$$

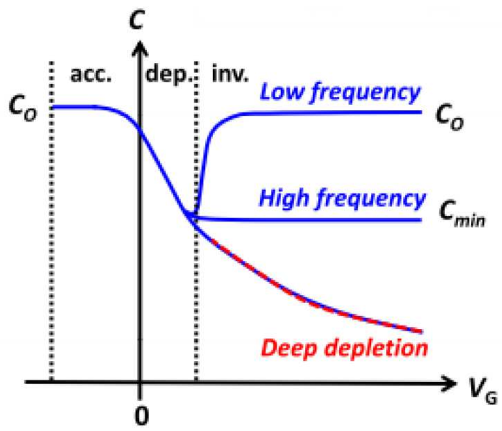
MOS Capacitance

$$C = \frac{C_{ox}\, C_j}{C_{ox} + C_j}\,, \qquad C_j : depletion\ capacitance$$

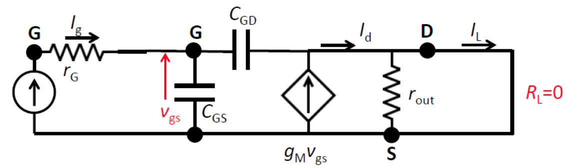
$$\frac{C}{C_{ox}} = \frac{1}{\sqrt{1 + \frac{2\,\varepsilon_{ox}^2\, V}{q\, N_A\, \varepsilon_S\, d}}}\,, \qquad C_{ox} : \frac{\varepsilon_{ox}}{d}$$

Accumulation: only majority carriers can respond to fast AC signal → added delta-charge

Deep depletion: DC bias changes so fast that minority carriers cannot respond. Therefore, the depletion layer keeps increasing



Cutoff frequency



$$I_d = g_m v_{gs}, \qquad I_g = \frac{v_{gs}}{1/j\omega C_{gt}}, \qquad C_{gt} = C_{gs} + C_{gd}$$

$$f_T = \frac{g_M}{2\pi(C_{gs} + C_{gd})} = \frac{3\mu_n}{4\pi}\,\frac{V_{GS} - V_T}{L^2}$$

Subthreshold swing

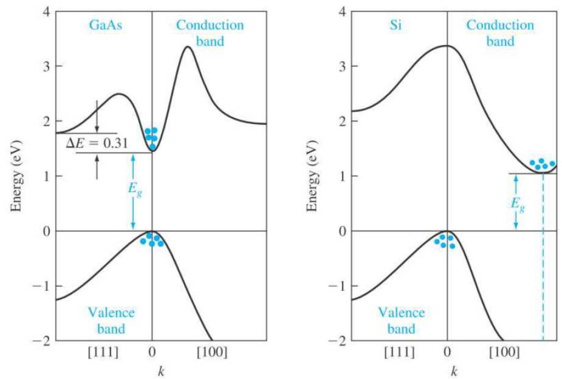
Subthreshold regime: $V_G < V_T$

Subthr. Swing: how effective can it be turned on / off

$$S = \frac{1}{\frac{d\ (\ln(I_D))}{d\ V_G}}$$

7. Various / General

Direct and Indirect Bandgaps



A transition in an indirect bandgap material must necessarily include an interaction with the crystal so that crystal momentum is conserved

Material properties

Selected Gate Materials

Gate Material	Work Function (eV)
n+ Polysilicon	4.0
Al	4.25
W	4.6
MoSi$_2$	4.5
PtSi	5.4
Pd$_2$Si	5.1

Selected Gate Insulators $\Delta E_C = \chi_S - \chi_i$

Insulator	ε_r	Gap (eV)	ΔE_C to Si
SiO$_2$	3.9	8-9	3.2
Si$_3$N$_4$	7.2-7.6	5.1	2.0
Al$_2$O$_3$	9.0	8.7	2.1
Ta$_2$O$_5$	26	4.5	0.5
ZrO$_2$	25	5.8	1.2
HfO$_2$	25	5.7	1.5
TiO$_2$	80	3.5	1.2

Tipps & Tricks

Energy: $E = \int q \cdot \varepsilon \cdot dx$

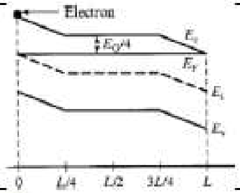
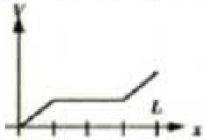
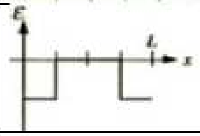
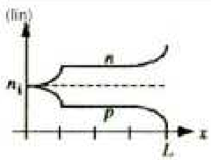
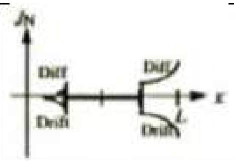
$E_{kin} + E_{pot} = const.$

E-Field: $\varepsilon = \frac{1}{q}\frac{dE}{dx} = -\frac{dV}{dx}$

Potential: $V = -\int \varepsilon \cdot dx \; = \; -\frac{1}{q}E_C$

Charge density: $\rho = \; \varepsilon_r \varepsilon_0 \frac{d\varepsilon}{dx}$

Charge d. with depletion approximation: $q \cdot (N_A - N_D)$

	
$V = -\frac{1}{q}E_c$	
$E_{pot} + E_{kin} = const$ $E_{pot} = E_c\ (electron)$ $E_{pot} = -E_c\ (hole)$	 
$\vec{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_C}{dx}$	
$n = n_i e^{-\frac{E_i - E_f}{kT}}$ $p = n_i e^{+\frac{E_i - E_f}{kT}}$ logarithmische Achse: $n_{\log} = \ln(n_i) + \frac{E_F - E_i}{kT}$ $p_{\log} = \ln(n_i) - \frac{E_F - E_i}{kT}$	
$J_{drift,n} = nq\mu_n\vec{E}$ $J_{diff,n} = q\frac{kT}{q}\mu_n\frac{dn}{dx}$	

Diamond structure