Electronic Circuits Summary

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Constants (@300K)

$$\varepsilon_0 = 8.854 * 10^{-12}\, F/m \qquad m_0 = 9.11 * 10^{-31}\, kg$$

$$k = 1.38 * 10^{-23}\, J/K = 8.617 * 10^{-5}\, eV/K$$

$$\frac{kT}{q} = 0.0259\, V\ ,\ \frac{q}{kT} = 38.61 \frac{1}{V}\ ,\ kT = 25.9\, meV$$

$$1\, eV = 1.602 * 10^{-19}\, J \qquad q = 1.602 * 10^{-19}\, A\, s$$

1. Transistor Characteristics

Resistor: $V_R = R * I_R$

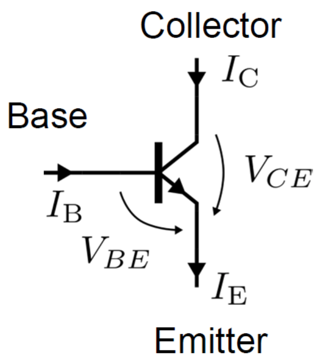
Capacitor: $I_C = C * \frac{d}{dt} V_C$

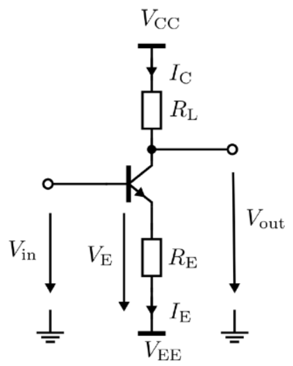
Inductor: $V_L = L \frac{d}{dt} I_L$

Bipolar Junction Transistor (BJT)

$$I_C = I_S * e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE}}{V_A} \right) , \qquad V_T = \frac{kT}{q} \approx 26\, mV$$

$$I_B = \frac{I_C}{\beta} , \qquad I_E = (1 + \beta)\, I_\beta , \qquad V_A : Early\ voltage$$





Small Signal Equivalent Circuit BJT

Consider only small oscillations around operation point

→ Linearize as approximation, $\boldsymbol{V_{CC} = 0 = V_{EE}}$ as const.

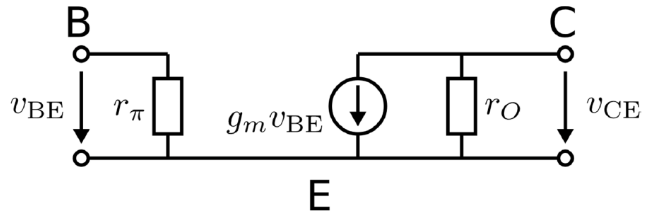
$$i_C = \frac{\beta}{\beta + 1} \frac{v_E}{R_E} = \frac{d\, I_C}{d\, V_{BE}} (v_{in} - v_E) = g_m * \Delta V_{BE} \approx \frac{v_E}{R_E}$$

$$v_{out} \approx - \frac{g_m R_L}{1 + g_m R_E} v_{in}$$

$$g_m = \frac{d\, I_C}{d\, V_{BE}} = \frac{I_C}{V_T} , \qquad g_\pi = \frac{d\, I_B}{d\, V_{BE}} = \frac{g_m}{\beta}$$

$$g_0 = \frac{d\, I_C}{d\, V_{CE}} = \frac{I_C}{V_A + V_{CE}} \approx \frac{I_C}{V_A}$$

$$r_\pi = \frac{1}{g_\pi} = \frac{\beta}{g_m} , \qquad r_0 = \frac{1}{g_0} = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$$



Biasing of a BJT (Setting the operationg point)

Voltage divider R_{B1}, R_{B2} sets the bias voltage

Transistor in active region : $V_{BE} \approx 0.7\, V$

MOSFET

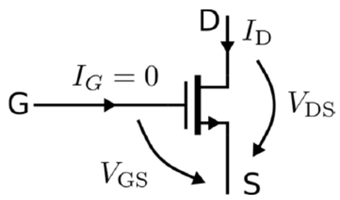
$$I_D = \frac{K'}{2} \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda\, V_{DS}), \qquad V_{DS} > V_{GS} - V_t$$

K' : Intrinsic transconduct. coeff.

V_t : Threshold voltage

W / L : Gate width / Gate length

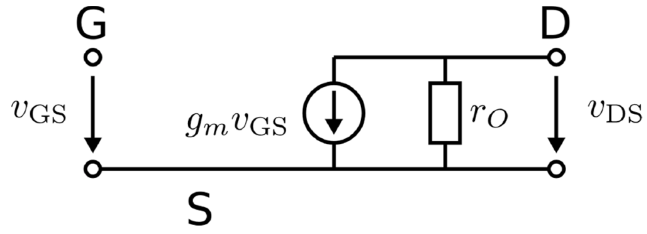
λ : Characteristic length



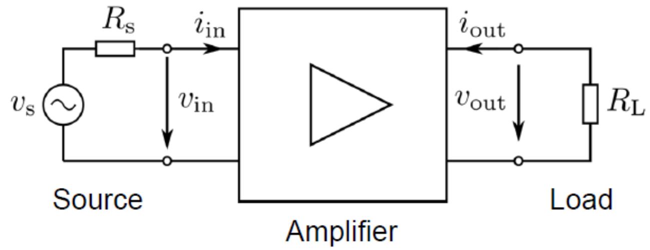
Small Signal Equivalent Circuit MOSFET

$$g_m = \frac{d\, I_D}{d\, V_{GS}} \approx \sqrt{\frac{2\, K'\, W}{L} I_D}$$

$$g_0 = \frac{d\, I_D}{d\, V_{DS}} = I_D \frac{\lambda}{1 + \lambda\, V_{DS}} \approx \lambda\, I_D , \qquad r_0 = \frac{1}{g_0} \approx \frac{1}{\lambda\, I_D}$$



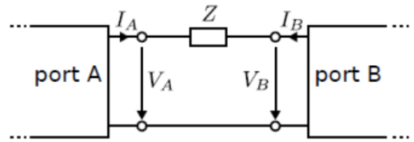
2. Single-Transistor Amplifiers



Impedances: $Z_{in} = \frac{v_{in}}{i_{in}}$, $Z_{out} = \frac{v_{out}}{i_{out}}$ ($v_S = 0$)

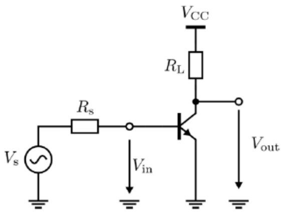
Gains: $A_V = \frac{v_{out}}{v_S}$, $A_I = \frac{i_{out}}{i_s}$ ($R_L = 0$)

Millers theorem



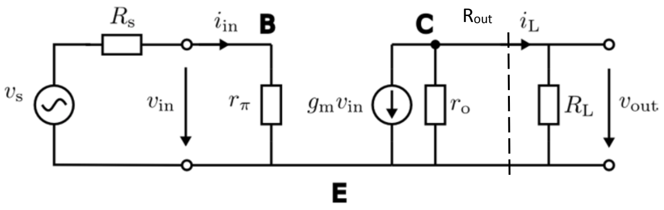
$$Z_{in} = \frac{Z}{1 + |A_V|} , \qquad Z_{out} = \frac{Z}{1 + \frac{1}{|A_V|}}$$

Common-Emitter / Source Amplifier



$$R_{in} = \frac{v_{in}}{i_{in}} = r_\pi$$

$$R_{out} = \frac{v_{out}}{i_L}\bigg|_{v_{in}=0} = r_0$$



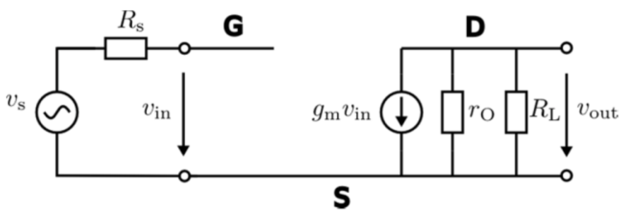
$$i_L = -\frac{r_0}{r_0 + R_L}\, g_m\, v_{in}$$

$$A_V = \frac{v_{out}}{v_S} \approx -\frac{r_\pi}{r_\pi + R_S}\, g_m\, R_L\,, \quad \frac{v_{out}}{v_{in}} = \frac{i_L\, R_L}{v_{in}} \approx -g_m\, R_L$$

Inverting Amplifier → 180° phase shift

$$A_I = \frac{i_{out}}{i_S}\bigg|_{v_{out}=0} = \frac{R_S}{R_S + r_\pi}\beta\,, \qquad \frac{i_{out}}{i_{in}}\bigg|_{v_{out}=0} = g_m\, r_\pi = \beta$$

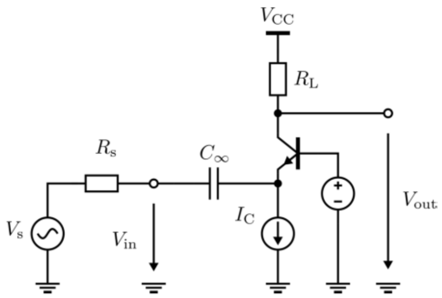
MOSFET: instead of BJT, no current into the gate

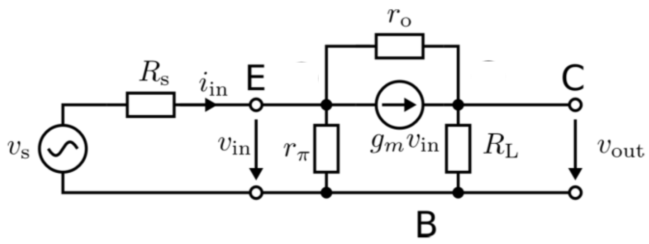


No current flow into the gate: $R_{out} \to \infty$, $v_{in} = v_S$

$$R_{out} = r_0\,, \qquad A_V \approx -g_m\, R_L$$

Common-Base / Gate Amplifier





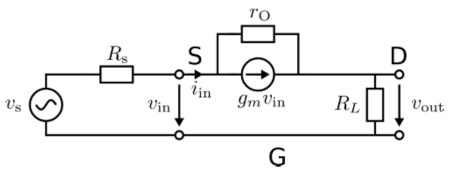
$$R_{in} = \frac{v_{in}}{i_{in}} \approx \frac{1}{g_m}$$

$$R_{out} = \frac{v_{out}}{i_{out}}\bigg|_{R_S \gg r_\pi} \approx ((r_\pi || R_S)\, g_m + 1)\, r_0 \approx \beta\, r_0$$

$$A_V = \frac{v_{out}}{v_S} \approx \frac{g_m\, R_L}{1 + g_m\, R_S} \quad : non-inverting\ amp.$$

$$A_I = \frac{i_{out}}{i_{in}}\bigg|_{v_{out}=0} = -\frac{g_m}{\frac{1}{R_S || r_\pi} + g_m} \approx -1 \quad for\ R_S \gg r_\pi$$

MOSFET: $R_S = 0 : voltage\ source\ ; R_S = \infty : current\ source$



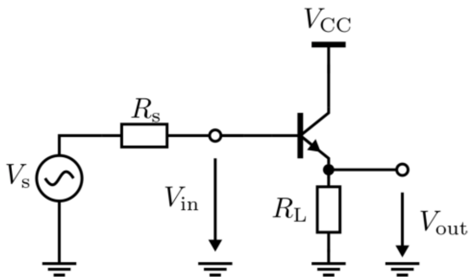
$$v_{out} \approx g_m R_L v_{in}$$

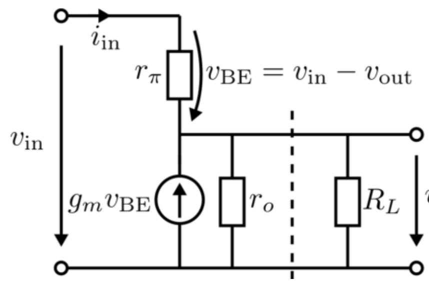
$$A_V = \frac{v_{out}}{v_S} \approx \frac{g_m R_L}{1 + g_m\, R_S}\,, \qquad A_I = \frac{i_{out}}{i_S} = -1$$

$$R_{in} = \frac{v_{in}}{i_{in}} \approx \frac{1}{g_m}\,, \qquad R_{out} \approx (1 + g_m R_S)\, r_0$$

Common-Collector / Drain Amplifier

Also known as emitter / source follower



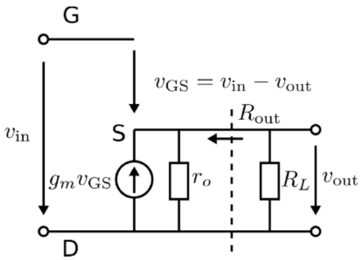


$$Usually: \quad R_L \ll r_0 \quad \to \quad R_L\, || r_0 \approx R_L$$

$$R_{in} = \frac{v_{in}}{i_{in}} = r_\pi + (\,1 + \beta)\, R_L\,, \qquad R_{out} = \frac{1}{g_m + \frac{1}{r_\pi}} \approx \frac{1}{g_m}$$

$$A_V = \frac{v_{out}}{v_{in}} \approx \frac{1}{1 + \frac{1}{g_m R_L}} \approx 1\,, \qquad A_I = 1 + \beta$$

MOSFET: no current flowing into the gate ($A_I = \infty$)



$$R_{in} = \infty$$

$$R_{out} = \frac{1}{g_m}$$

$$A_V \approx \frac{1}{1 + \frac{1}{g_m\, R_L}}\,, \qquad i_{out} = g_m\, v_{out}$$

Comparison of the three basic amplifiers

	CE/CS Amplifier	CC/CD Amplifier (Voltage Buffer)	CB/CG Amplifier (Current Buffer)
Voltage Gain $\lvert A_V \rvert$	High	~1	High
Current Gain $\lvert A_I \rvert$	High	Moderate	~1
Input Resistance R_{in}	High	High	Low
Output Resistance R_{out}	High	Low	High

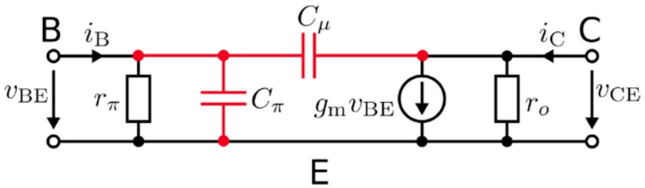
After voltage buffer: *lower* output resistance (better V-source)
After current buffer: *larger* output resistance (better C-source)

Impedance Matching

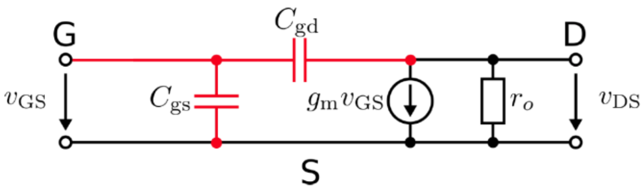
$$P_L \text{ is maximized when } R_L = R_S$$

3. Frequency Response of Amplifiers

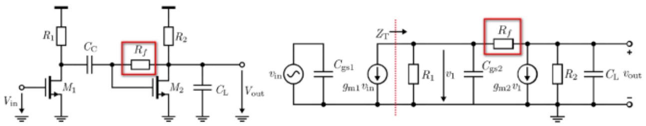
Change of charge vs. voltage across pn-junctions between BJTs can be represented by a parasitic capacitance



$$C_\pi : \text{capacitance between B and E}, \quad \triangleq C_{gs}$$
$$C_\mu : \text{capacitance between B and C}, \quad \triangleq C_{gd}$$

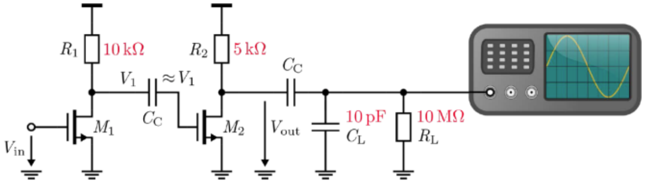


Bandwidth-Broadening



Additional **shunt/feedback resistor R_f** up to doubles bandwidth!

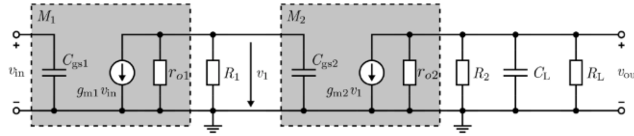
Multi-Stage Amplifier



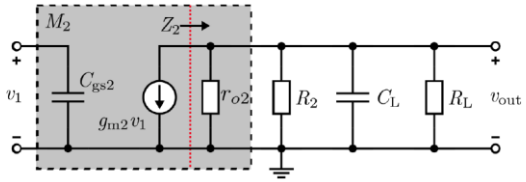
Small signal model: all constant voltage supplies become ground

$$C_C : \text{large}, \approx \text{shorts}, \qquad C_{gd} : \text{often negligible}$$

Stage 1:



Stage 2:

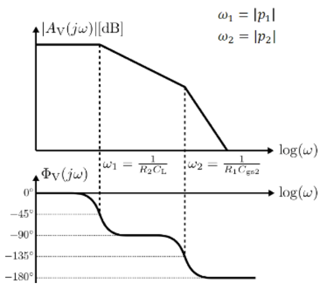


$$v_{out} = -g_{m2}\, v_1\, Z_2 \approx \frac{-\,g_{m2}\, R_2}{1 + sC_L R_2}\, v_1\,, \qquad Z_2 \approx \frac{R_2}{1 + sC_L R_2}$$

$$A_{V2}(s) = \frac{v_{out}(s)}{v_1(s)} = -\frac{g_{m2}\, R_2}{1 + s\, C_L\, R_2}$$

Cut-off frequencies: defined by poles

$$A_V(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{g_{m1} R_1 * g_{m2} R_2}{(1 + s\, R_1\, C_{gs2})(1 + s\, R_2\, C_L)}$$



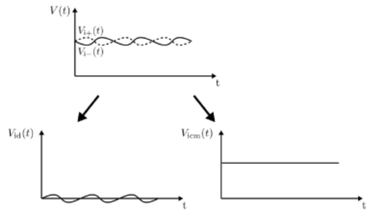
$$\omega_1 = |p_1| = \frac{1}{R_2 C_L} \quad \omega_2 = |p_2| = \frac{1}{R_1 C_{gs2}}$$

Time Domain representation:

$$v_{out}(t) = A * e^{p_1 t} + B * e^{p_2 t} + C$$

4. Differential Amplifiers

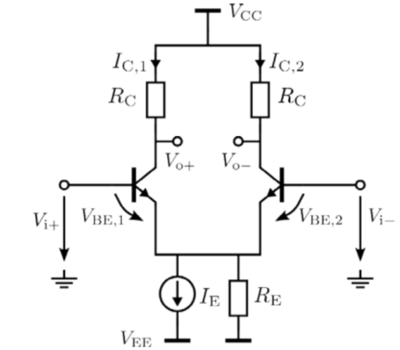
Transmitting information with two complementary signals
Information is contained in the difference, same DC value

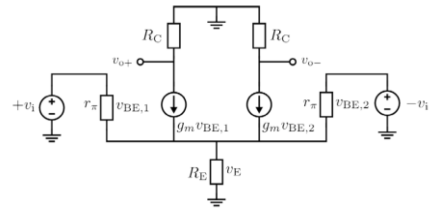


$$V_{id} = V_{i+} - V_{i-} \quad \text{Differential component}$$

$$V_{icm} = \frac{V_{i+} + V_{i-}}{2} \quad \text{Common-mode (DC component)}$$

Differential amplifier: In order to filter out DC component before the amplification, we use a fixed *tail current* I_E, which also enables *DC coupling of stages* (current splitted)





V_E (emitter-node potential) remains constant → $v_E = 0$
Symmetry between left and right branch → split circuit into two independent parts and analyze separately (once)

Differential amplification: $A_{vd} = \frac{v_{od}}{v_{id}} = -g_m R_C$

Common-mode amplification: $A_{vcm} = \frac{v_o}{v_i} = \frac{v_{ocm}}{v_{icm}} = -\frac{R_C}{2R_E}$

Common Mode Rejection Ratio

Indicates how strong a common mode signal is attenuated compared to a differential signal

$$G = \frac{A_{vd}}{A_{vcm}} = \frac{-g_m\,R_C}{-\,R_C/2R_E} = 2\;g_m\,R_E$$

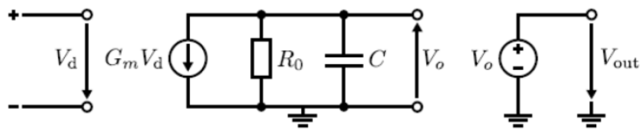
$$CMRR = G_{dB} = 20 * \log_{10} G$$

$$GBP = A_0 * \omega_C$$

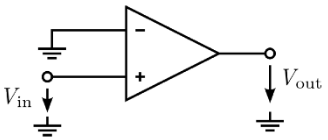
Operational amplifiers

Ideal: $Z_{in} \to \infty$, $Z_{out} \to 0$, $A_{vd} \to \infty$, $CMMR \to \infty$

Non-ideal Small Signal Equivalent

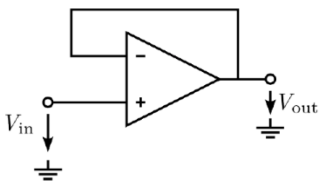


Voltage Comparator



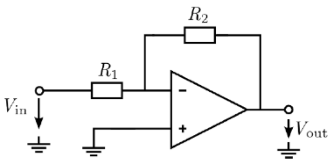
$$V_{out} = sign(V_{in}) * V_{CC}$$

Voltage follower (Buffer)



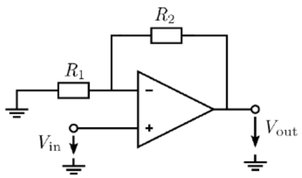
$$V_{\text{out}} = V_{\text{in}}$$

Inverting amplifier



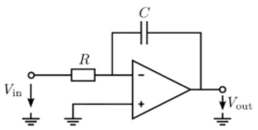
$$V_{\text{out}} = -\frac{R_2}{R_1} V_{\text{in}}$$

Non-inverting amplifier



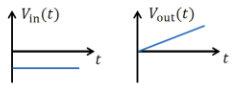
$$V_{\text{out}} = \left(1 + \frac{R_2}{R_1}\right) V_{\text{in}}$$

Integrator

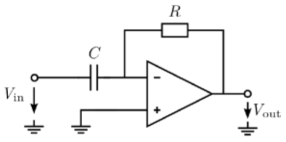


$$V_{\text{out}}(s) = -\frac{V_{\text{in}}(s)}{sRC}$$

$$V_{\text{out}}(t) = V_{\text{out}}(0) - \frac{1}{RC}\int_0^t V_{\text{in}}(t)d\tau$$

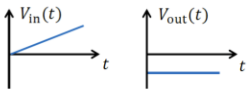


Differentiator



$$V_{\text{out}}(s) = -V_{\text{in}}(s)RCs$$

$$V_{\text{out}}(t) = -RC\frac{d}{dt}V_{\text{in}}(t)$$



5. Instrumentation Amplifier

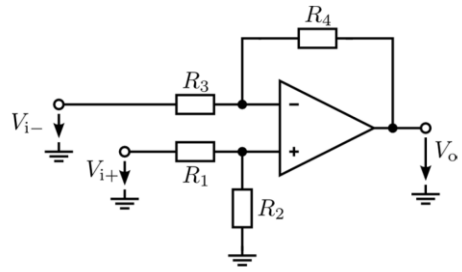
Precise amplification of weak, distorted sensor signals

High input impedance, internal feedback loop

Basic Instrumentation Amplifier

Amplifies voltage difference with a precise gain

Differential gain must be equal for both input branches



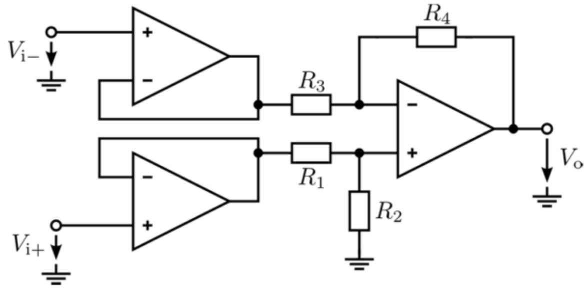
$$V_0 = \frac{R_2}{R_1}\frac{1 + R_4/R_3}{1 + R_2/R_1}\,V_{i+} - \frac{R_4}{R_3}\,V_{i-}$$

Set $R_1 = R_3$, $R_2 = R_4$ to equally load both input branches:

$$V_0 = G * V_{i+} - G * V_{i-}\,, \qquad G = \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

Buffered Instrumental Amplifier

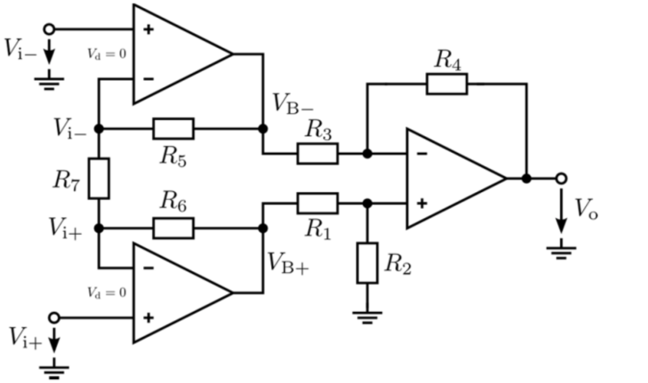
Obtain ideally high input impedance by input buffering



$$V_0 = V_{icm}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} - \frac{R_4}{R_3}\right) + \frac{V_{id}}{2}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} + \frac{R_4}{R_3}\right)$$

$$CMMR = \frac{A_d}{A_{cm}} = \frac{V_0/V_{id}}{V_0/V_{icm}} = \frac{(R_3 + R_4)R_2 + (R_1 + R_2)R_4}{2 * (R_2R_3 - R_4R_1)}$$

Input stage gain



Differential & common mode gain of **input stage**:

$$A_B = \frac{V_{Bd}}{V_{id}} = \frac{V_{B+} - V_{B-}}{V_{i+} - V_{i-}} = 1 + \frac{R_5 + R_6}{R_7}$$

$$A_{cm,B} = \frac{V_{B+} + V_{B-}}{V_{i+} + V_{i-}} = 1 \rightarrow no\ current\ through\ R_5, R_6, R_7$$

Differential & common mode gain in **total**:

$$A'_d = \frac{V_0}{V_{id}} = \frac{A_B}{2}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} + \frac{R_4}{R_3}\right)$$

$$R_1 = R_3, R_2 = R_4 \quad \rightarrow \quad A'_d = \frac{R_2}{R_1} A_B$$

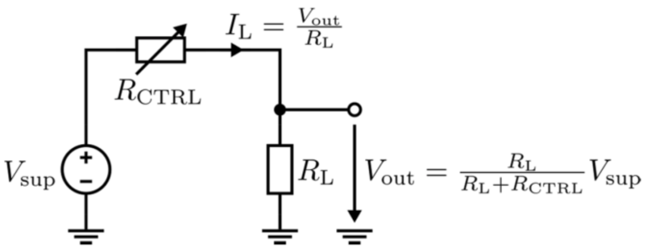
$$A_{cm} = A_{cm,B}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} - \frac{R_4}{R_3}\right)$$

$$CMMR = \frac{A'_d}{A_{cm}} = A_B \frac{A_d}{A_{cm}} \quad \rightarrow \quad increased\ by\ factor\ A_B$$

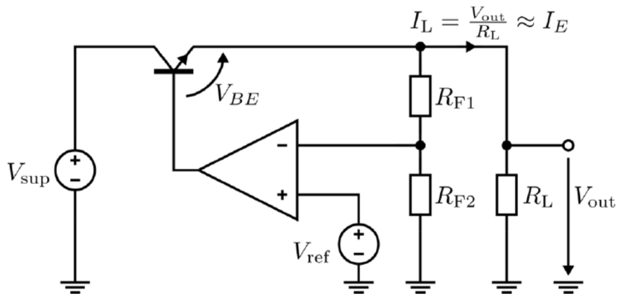
Voltage offset: Offset voltage in combination with a small input signal is highly undesired. The output signal then reaches the saturation level even for small values of V_i and is therefore distorted.
If this is not appropriate, **chopper amplifiers** can be used.

6. Voltage Regulators, Logarithmic & Anti-Logarithmic Amplifiers

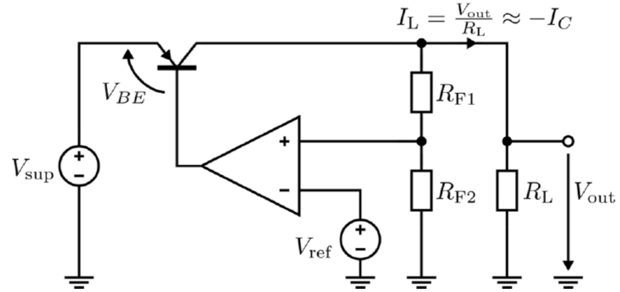
Linear voltage regulators



Non-inverting topology



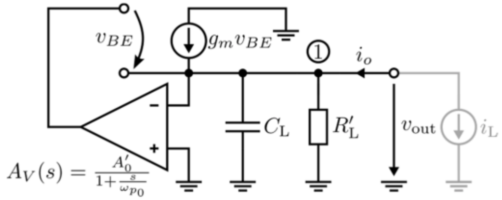
Inverting topology



$$V_{out} = \left(1 + \frac{R_{F1}}{R_{F2}}\right) V_{ref}$$

$$Choose \quad R_{F1}, R_{F2} \gg R_L \quad \rightarrow \quad I_L \approx I_E \,/\, -I_C$$

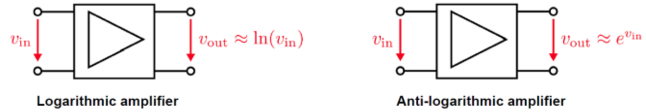
Small Signal Equivalent:



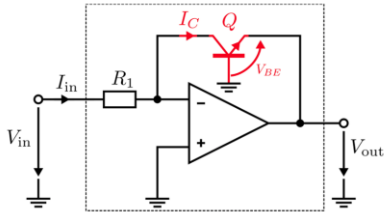
$$Z_{out} = \frac{v_{out}}{i_o} = \frac{1}{g_m A'_0} \frac{1 + \frac{s}{\omega p_0}}{\left(1 + \frac{s}{\omega p_0 A'_0}\right)\left(1 + \frac{s\, C_L}{g_m}\right)}$$

Logarithmic & Anti-Logarithmic Amplifiers

Non-linear circuit whose output voltage is proportional to the logarithm / exponential of the input voltage

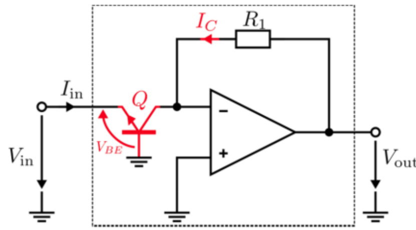


Logarithmic Amplifier: Rely on logarithmic relationship of I_C & V_{BE}



$$I_{in} = \frac{V_{in}}{R_1} = I_C = I_S\, e^{-\frac{V_{out}}{V_T}}, V_{out} = -V_T \ln\left(\frac{I_{in}}{I_S}\right) = -V_T \ln\left(\frac{V_{in}}{R_1 I_S}\right)$$

Anti-Logarithmic Amplifier



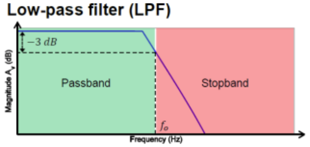
$$V_{BE} = -V_{in}, I_C = I_S e^{-\frac{V_{in}}{V_T}} \quad \rightarrow V_{out} = I_C\, R_1 = I_S\, R_1\, e^{-\frac{V_{in}}{V_T}}$$

7. Active RC Filters

Filter is a frequency-selective circuit that passes a specified band of frequencies and blocks frequencies outside of it.

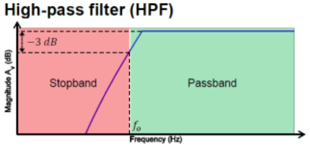
Passive Filters: based on passive elements such as R / L / C
Active Filters: based on op-amps in addition to R / L / C

Low-pass filter (LPF)



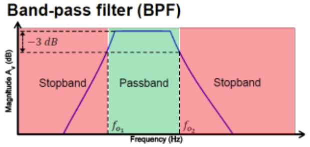
LPF passes frequencies from DC to a desired frequency f_o.

High-pass filter (HPF)



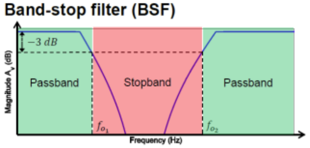
HPF passes frequencies starting from a desired frequency f_o.

Band-pass filter (BPF)



BPF passes a band of frequencies from f_{o_1} to f_{o_2} and stops all other frequencies.

Band-stop filter (BSF)



BSF blocks a band of frequencies from f_{o_1} to f_{o_2} and passes all the other frequencies.

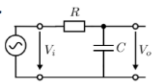
Cutoff frequency

Poles define cut-off: $\quad |T(j\omega_c)| = \frac{1}{\sqrt{2}} \max(|T(j\omega)|)$

$$\omega_n = |p_n|, \qquad BW_{-3dB} = {}^{\omega_0}/_{Q_0}$$

First order passive filters

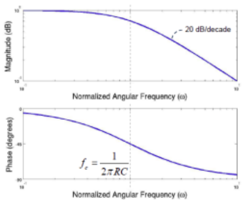
Low-pass Filter



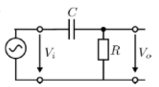
Transfer function: $T(s) = \frac{V_o}{V_i} = \frac{1}{1+sRC}$

Amplitude response: $\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$

Phase response: $\angle\left(\frac{V_o(j\omega)}{V_i(j\omega)}\right) = -\tan^{-1}(\omega RC)$



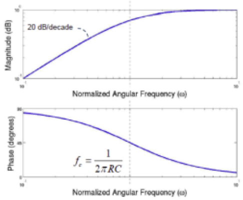
High-pass Filter



$T(s) = \frac{V_o}{V_i} = \frac{sRC}{1+sRC}$

$\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$

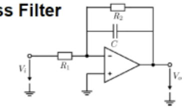
$\angle\left(\frac{V_o(j\omega)}{V_i(j\omega)}\right) = 90^\circ - \tan^{-1}(\omega RC)$



First order active filters

Filters and amplifies signal

Low-pass Filter

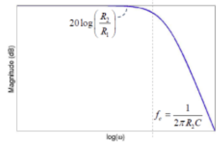


Transfer function: $T(s) = \frac{V_o}{V_i} = -\frac{R_2}{R_1}\cdot\frac{1}{1+sR_2C}$

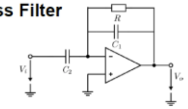
Amplitude response: $|T(j\omega)| = \frac{R_2}{R_1}\cdot\frac{1}{\sqrt{1+(\omega R_2 C)^2}}$

$|T(j\omega=0)| = \frac{R_2}{R_1}$

Phase response: $\angle T(j\omega) = 180^\circ - \tan^{-1}(\omega R_2 C)$



High-pass Filter

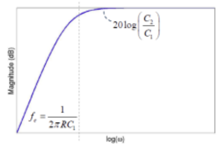


$T(s) = \frac{V_o}{V_i} = -\frac{sRC_2}{1+sRC_1}$

$|T(j\omega)| = \frac{\omega RC_2}{\sqrt{1+(\omega RC_1)^2}}$

$|T(j\omega\to\infty)| = \frac{C_2}{C_1}$

$\angle T(j\omega) = -90^\circ - \tan^{-1}(\omega RC_1)$

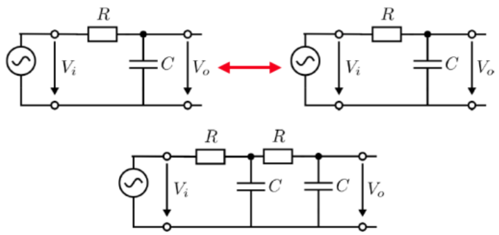


Comparison of first order filters

Passive Filters	Active Filters
2 passive elements that determine pole	3 passive elements, pole determined by elements in op-amp feedback
Fixed gain of 1 in pass band	Variable pass band gain possible
No power consumption	Op-amp consumes power
Real filter transfer function close to ideal filter function	Real filter transfer function dependent on op-amp DC-gain and gain-bandwidth product

Second order passive filters

2^{nd} order passive filters can be synthesized from 1^{st} order



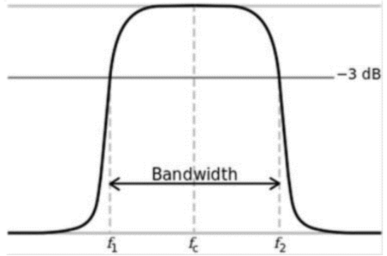
$$T(s) = \frac{\frac{1}{R^2C^2}}{s^2 + \frac{3}{RC}\,s + \frac{1}{R^2C^2}}, \qquad \omega_0 = \frac{1}{RC}, Q_0 = \frac{1}{3}$$

Denominator: $\quad D(s) = s^2 + \frac{\omega_0}{Q_0}\,s + \omega_0^2$

Poles: $\quad p_{1,2} = -\frac{\omega_0}{2\,Q_0} \pm \omega_0\sqrt{\frac{1}{4\,Q_0} - 1}$

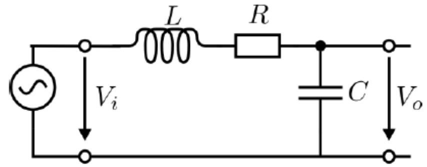
Resonance frequency: $\quad \omega_0 = 1/\sqrt{L\,C}$

Quality factor: $\quad Q_0 = \frac{1}{R}\sqrt{{}^{L}/_{C}}$



Energy vs. freq.
The higher Q, the narrower and sharper the peak.

Low-pass filter

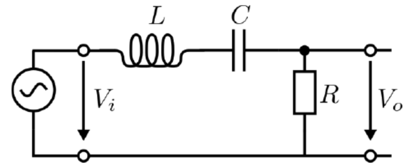


$$T(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}\,s + \frac{1}{LC}} = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q_0}\,s + \omega_0^2}$$

High-pass filter

$$T(s) = \frac{s^2 * \omega_0^2}{s^2 + \frac{\omega_0}{Q_0}\,s + \omega_0^2}$$

Band-pass filter

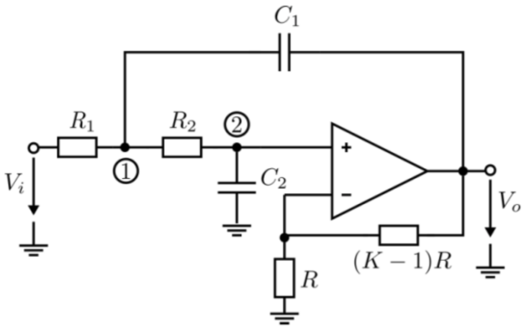


$$T(s) = \frac{s\frac{R}{L}}{s^2 + \frac{R}{L}\,s + \frac{1}{LC}}$$

Sallen-Key amplifier

Allow sharp gains without using inductors (expensive)

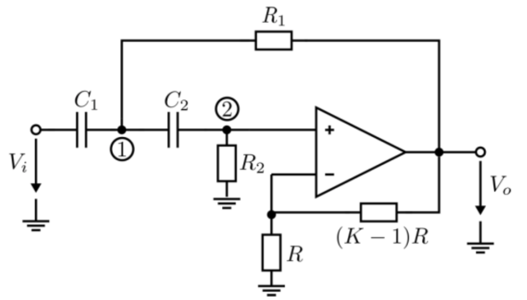
Low-pass filter



$$T(s)=\frac{\frac{K}{R_1R_2C_1C_2}}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}\qquad Q_0=\frac{\omega_0}{\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right)}$$

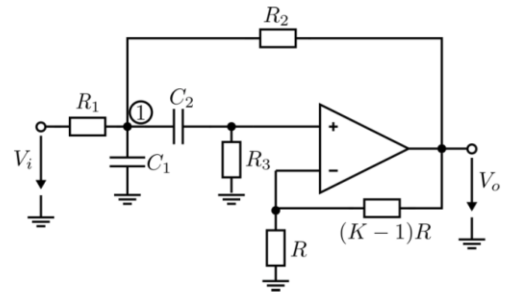
High-pass filter



$$T(s)=\frac{Ks^2}{s^2+\left(\frac{1}{R_2C_2}+\frac{1}{R_2C_1}+\frac{1-K}{R_1C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}\qquad Q_0=\frac{\omega_0}{\left(\frac{1-K}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}\right)}$$

Band-pass filter

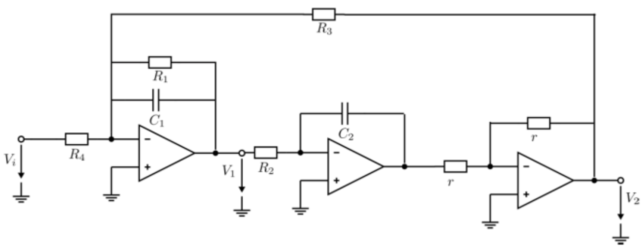


$$T(s)=\frac{\frac{K}{R_1C_1}s}{s^2+\left(\frac{1}{R_1C_1}+\frac{1-K}{R_2C_1}+\frac{1}{R_3C_1}+\frac{1}{R_3C_2}\right)s+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

$$\omega_0=\sqrt{\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}\qquad Q_0=\frac{\omega_0}{\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_2}+\frac{1}{R_3C_1}+\frac{1-K}{R_2C_1}\right)}$$

Tow-Thomas Biquad filter

Less sensitive to tolerance difference; combine Sallen-Keys



$$T_{LP}(s)=\frac{V_2}{V_i}=-\frac{\frac{1}{R_2R_4C_1C_2}}{s^2+\frac{1}{R_1C_1}s+\frac{1}{R_2R_3C_1C_2}}$$

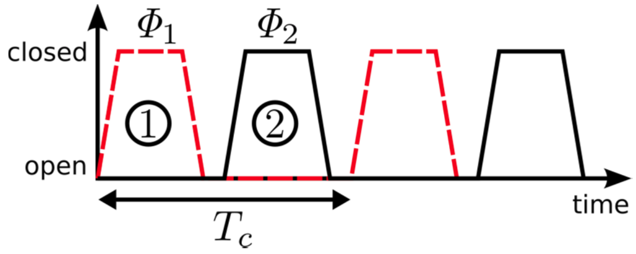
$$T_{BP}(s)=\frac{V_1}{V_i}=-\frac{\frac{1}{R_4C_1}s}{s^2+\frac{1}{R_1C_1}s+\frac{1}{R_2R_3C_1C_2}}$$

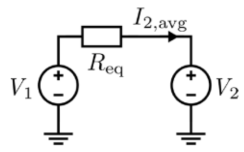
$$\omega_0^2=\frac{1}{R_2R_3C_1C_2}\qquad Q_o=\omega_0R_1C_1$$

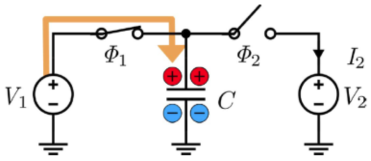
8. Switched capacitor filters

Motivation: some systems require an active RC low-pass filter with very low f_{cutoff} → We need a large resistor in a highly integrated chip, whereby it is also inaccurate.

Concept of switched capacitor devices







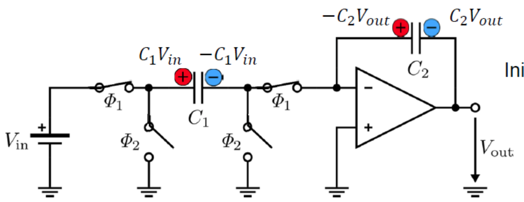
Transfer of charge ΔQ from potential V_1 to potential V_2 at a fixed rate $f_c=\frac{1}{T_C}$.

Transferred charge per T_C: $\Delta Q = C\,(V_1-V_2)$

Average current: $I_{2,avg}=\frac{\Delta Q}{T_C}=\frac{C\,(V_1-V_2)}{T_C}$

Equivalent resistor: $R_{eq}=\frac{T_C}{C}=\frac{1}{f_C\,C}$

Inverting Integrator using SC

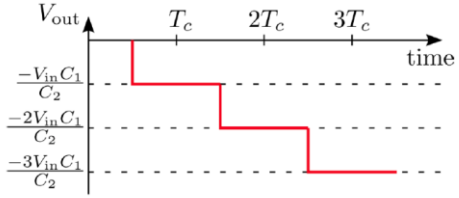


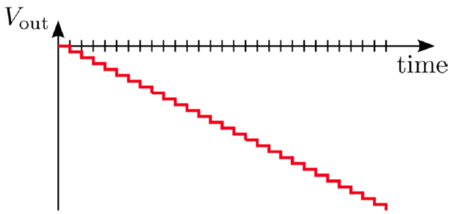
Initial condition:
$V_{out} = 0$

Phase 1: Φ_1 *on , charge accumulates on* C_1 *and* C_2

Phase 2: Φ_2 *on,* C_1 *is discharged*

$$C_2\, V_{out}[nT_C] = C_2\, V_{out}[(n-1)T_C] - C_1\, V_{in}[n\,T_C]$$





$\Longrightarrow$ seems continuous for small enough T_C

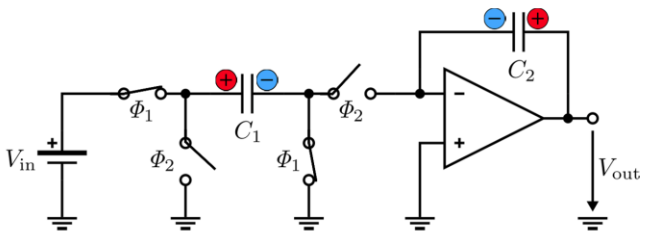
$$V_{out}[n\,T_C] = -\frac{C_2}{T_C C_2}\int_0^{n\,T_C} V_{in}(t)\,dt$$

Ratio of capacitors can be realized more accurate than absolute values of R and C.

Z-transform

Definition	$Z\{x[nT_c]\} = X(z)T = \sum_{k=-\infty}^{\infty} x[kT_c]z^{-k}$
Time delay	$Z\{x[(n-k)T_c]\} = z^{-k}X(z)$
Mapping to Laplace domain	$s = \frac{z-1}{T_c}$ or $s = \frac{1-z^{-1}}{T_c}$ (forward/backward Euler transform)
Mapping to $j\omega$-axis	$z = e^{j\omega T_c} = e^{j\frac{2\pi f}{f_c}}$

Non-inverting Integrator using SC



Same circuit as before, only **change of switching schedule**
$\Longrightarrow$ ***Charge on C_2 is inverted*** *compared to before*

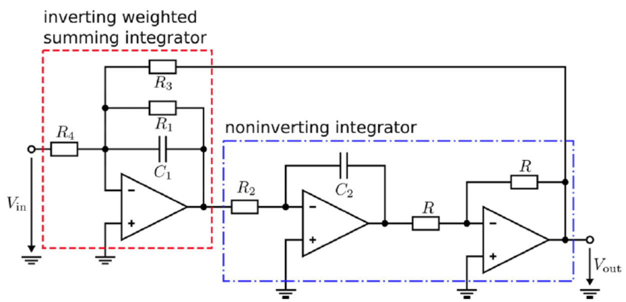
Phase 1: C_1 *is charged to* V_{in}

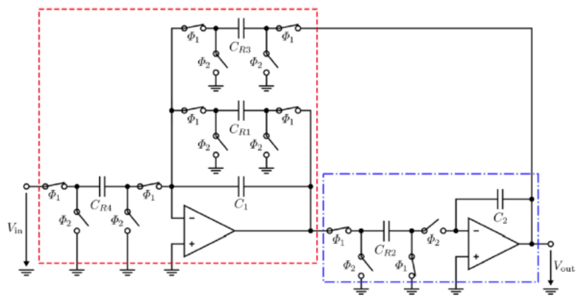
Phase 2: *Charge is transferred to* C_2

$$C_2\, V_{out}[n\,T_C] = C_2\, V_{out}[(n-1)T_C] + C_1\, V_{in}[n\,T_C]$$
$$C_1\, V_{in}(z) = C_2\,(1 - z^{-1})\, V_{out}(z)$$

$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{C_1}{C_2}\frac{1}{1-z^{-1}}$$

Switched capacitor Tow-Thomas biquad





$\Longrightarrow$ Replace all reistors by switched capacitors

$$T(z) = \frac{V_{out}(z)}{V_{in}(z)} = -\frac{C_{R4}}{C_{R3}}\frac{\frac{C_{R2}C_{R3}}{C_1C_2}}{z^{-2}+z^{-1}\left(-2-\frac{C_{R1}}{C_1}\right)+1+\frac{C_{R1}}{C_1}+\frac{C_{R2}C_{R3}}{C_1C_2}}$$

$$T(s) = \frac{k\omega_0^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \to T(z) \approx \frac{k(\omega_0T_c)^2}{z^{-2}+z^{-1}\left(-2-\frac{\omega_0T_c}{Q}\right)+1+\frac{\omega_0T_c}{Q}+(\omega_0T_c)^2}$$

Design equations:

$$\frac{C_{R4}}{C_{R3}} = -k\,, \qquad \frac{C_{R2}}{C_2} = \frac{C_{R3}}{C1} = \omega_0\,T_C\,, \qquad \frac{C_{R3}}{C_{R1}} = Q$$

Z-transform extended

Time discrete equivalent of the Laplace function.

$$X(z) = \sum_{k=-\infty}^{\infty} x[k]z^{-k}$$

Delay by n samples:

$$x[k] \to x[k-n] \quad \Leftrightarrow \quad X[z] \to z^{-n}X[z]$$

Transformation to frequency:

$$z = e^{i\omega T_C} = e^{\frac{i2\pi f}{f_C}}$$

Differentiation:

$$Z\left(\frac{df(t)}{dt}\right) \Rightarrow F(z)\frac{1-z^{-1}}{T_C}$$

Integration:

$$Z\left(\int f(t)dt\right) \Rightarrow F(z)\frac{T_C}{1-z^{-1}}$$

Forward Euler Transform: $s = \frac{z-1}{T_C}$

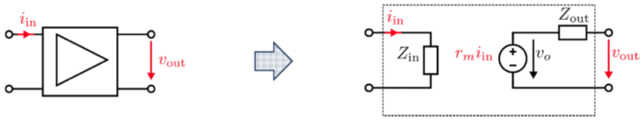
Backward Euler Transform : $s = \frac{1-z^{-1}}{T_C}$

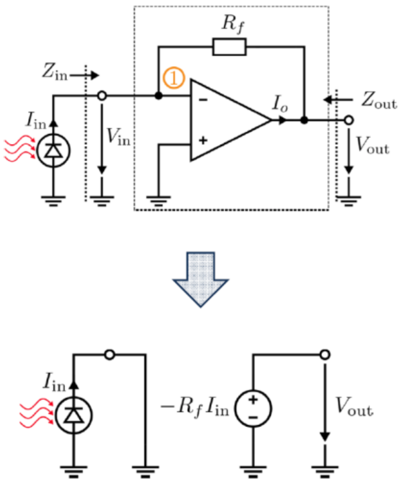
9. Appendix

Transimpedance amplifiers

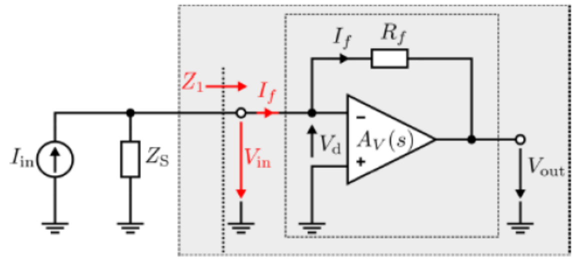
Sensing an input current & converting it to output voltage

$$r_m = \frac{dV_0}{dI_{in}}, \qquad Z_{in} \to 0, \qquad Z_{out} \to 0$$





Frequency Response of Transimpedance Amplifiers



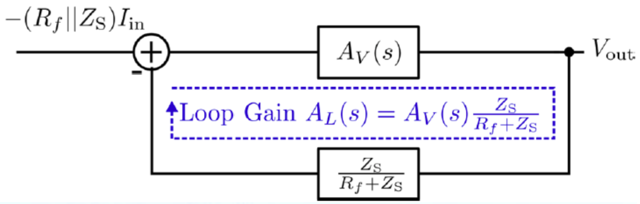
Transimpedance experiences broadbanding due to feedback:

$$Z_T' = \frac{V_{out}}{I_f} = -A_v(s)Z_1 = \frac{R_f A_0}{1 + A_0 + \frac{s}{\omega_{p_0}}} \approx \frac{R_f}{1 + \frac{s}{A_0\ \omega_{p_0}}}$$

Bandwidth is limited by GBP of op-amp.

Loop gain: $A_L(s) = A_V(s)\frac{Z_s}{R_f+Z_s}$

Feedback-factor: $\beta(s) = \frac{Z_s}{R_f+Z_s}$



→ **High bandwith trade-off**: High transimpedance gain results in lower bandwidth

Due to the capacitance $C_s' = C_s + C_{in}$ the transimpedance amplifier becomes a second-order system with DC transimpedance gain $\approx -R_f$ and a loop gain

$$A_L(s) = A_V(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p_0}}\right)\left(1+\frac{s}{\omega_{p_s}}\right)}$$

Step Response of Second-Order Systems

$$Z_T(s) = \frac{V_{out}}{I_{in}} = -R_f\frac{\omega_n^2}{(s-p_1)(s-p_2)} = $$
$$= -R_f\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Characteristic equation: $\boldsymbol{s^2 + 2\zeta\omega_n s + \omega_n^2 = 0}$ **:**

$\omega_n = \sqrt{A_0\omega_{p_0}\omega_{p_s}} \qquad \zeta = \frac{\omega_{p_0}+\omega_{p_s}}{2\sqrt{A_0\omega_{p_0}\omega_{p_s}}}$

$p_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}$

$p_1 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}$

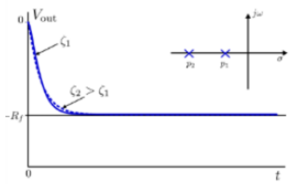
$\omega_{p_0} = -p_0 \qquad A_0\omega_{p_0} = GBP$

$\omega_{p_s} = \frac{1}{R_f C_s'} \qquad Q = \frac{1}{2\zeta}$

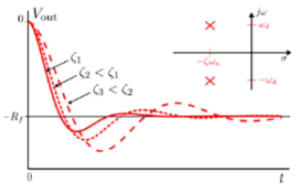
System behavior:

Overdamped ($\zeta > 1$): p_1, p_2 are real and negative
Critically damped ($\zeta = 1$): $p_1 = p_2 = -\omega_n$
Underdamped ($0 < \zeta < 1$): p_1, p_2 are complexe conjugates

Overdamped

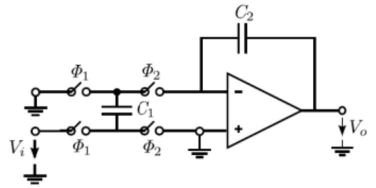


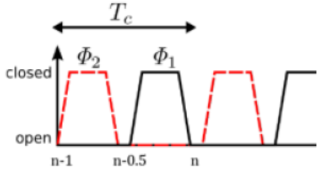
Underdamped



Switched Capacitors Examples

Example:

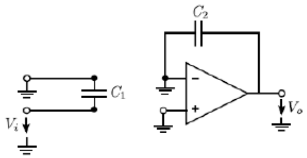




At the end of $\Phi_1(n-1)$:

$Q_{C1} = C_1 \cdot V_i(n-1)$

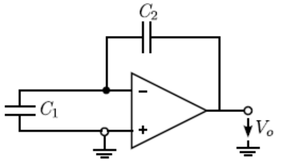
$Q_{C2} = C_2 \cdot V_0(n-1)$



At the end of $\Phi_2(n-0.5)$:

$Q_{C1} = C_1 \cdot 0$

$Q_{C2} = C_2 \cdot V_0(n-0.5)$



Charge conservation:

At $(n-0.5)$:

$C_2V_0(n-0.5) = C_2V_0(n-1) + C_1V_i(n-1)$

At (n): (nothing happens to Q_{C2})

$C_2V_0(n) = C_2V_0(n-0.5)$

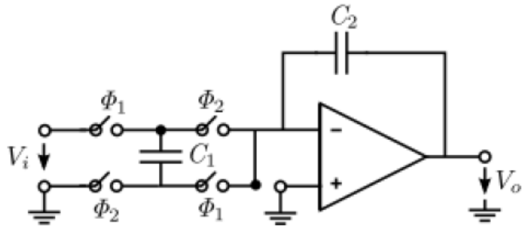
Results in:

$C_2V_0(n) = C_2V_0(n-1) + C_1V_i(n-1)$

$C_2V_0(Z) = C_2Z^{-1}V_0(Z) + C_1Z^{-1}V_i(Z)$

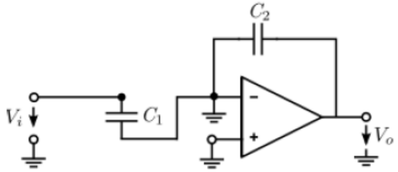
$$\frac{V_0(Z)}{V_i(Z)} = \frac{C_1}{C_2}\frac{Z^{-1}}{[1-Z^{-1}]}$$

Example 2:



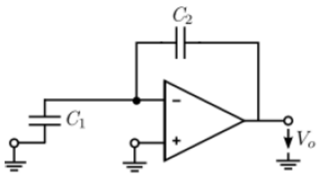
@ $\Phi_1(n-1)$:

$Q_1 = C_1V_i(n-1) \qquad Q_2 = C_2V_0(n-1)$



@ $\Phi_1(n-0.5)$:

$Q_1 = 0 \qquad Q_2 = C_2V_0(n-0.5)$



Charge conservation:

Überlegen wo positive und negative Ladung hingeht.
Verstärker verstärkt solange bis die eingangsspannungsdifferenz 0 ist.

@ (n-0.5):

$$C_2V_0(n-0.5) = C_2V_0(n-1) - C_1V_i(n-1)$$

@ (n):

$$C_2V_0(n) = C_2V_0(n-0.5) - C_1V_i(n)$$

$$\Rightarrow C_2V_0(n) = C_2V_0(n-1) - C_1V_i(n-1) - C_1V_i(n)$$

$$\Rightarrow \frac{V_0(z)}{V_i(z)} = -\frac{C_1}{C_2}\frac{1+Z^{-1}}{1-Z^{-1}}$$